Vidyamandir Classes
Gurukul for IITJEE & Medical Preparation

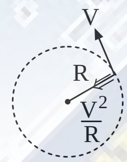
SOLUTIONS

2018-JEE Entrance Examination - Advanced | Paper-1

PART-I	PHYSICS

1.(BC) $U(r) = \dfrac{kr^2}{2}$

$F = -\dfrac{dU}{dr} = -kr \quad \Rightarrow \quad \dfrac{mv^2}{R} = kR \quad \Rightarrow \quad v = \sqrt{\dfrac{k}{m}}\,R$



$\Rightarrow \; L = m\,v\,R = \sqrt{mk}\,R^2$

2.(AC) $\bar{a} = \dfrac{\bar{F}}{m} = \dfrac{(1)\,t\,\hat{i} + (1)\,\hat{j}}{(1)} = t\,\hat{i} + \hat{j}$

$\Rightarrow \; \bar{v} = \int_0^t \bar{a}\,dt = \dfrac{t^2}{2}\hat{i} + t\hat{j} \quad \Rightarrow \quad \bar{r} = \int_0^t \bar{v}dt = \dfrac{t^3}{6}\hat{i} + \dfrac{t^2}{2}\hat{j}$

At $t = 1s \;\Rightarrow\; \bar{r} = \dfrac{1}{6}\hat{i} + \dfrac{1}{2}\hat{j}$, $\bar{F} = (1)\hat{i} + (1)\,\hat{j}$

$\bar{\tau} = \bar{r} \times \bar{F} = \dfrac{1}{6}\hat{k} - \dfrac{1}{2}\hat{k} = -\dfrac{1}{3}\hat{k}$

$\Rightarrow \; |\tau| = \dfrac{1}{3}$. Hence "A" is correct $\quad \Rightarrow \quad \bar{\tau}$ is along $-\hat{k}$. Hence "B" is incorrect

$\Rightarrow \; \bar{v}(t = 1s) = \dfrac{1}{2}\hat{i} + 1\hat{j}$. Hence "C" is correct

$\Rightarrow \; |\bar{s}| = |\bar{r}| = \dfrac{1}{6}\sqrt{10}m$. Hence "D" is incorrect.

3.(AC) $h = \dfrac{2\sigma\cos\theta}{\rho r g}$

$h \propto \dfrac{1}{r} \quad \Rightarrow \quad$ "A" is correct

$h \propto \sigma \quad \Rightarrow \quad$ "B" is incorrect

If lift is going up $\;\Rightarrow\; g_{eff} = g + a$

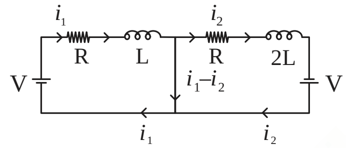
$\Rightarrow \; h = \dfrac{2\sigma\cos\theta}{\rho\,r(g + a)}$

$h \propto \dfrac{1}{g + a} \quad \Rightarrow \quad$ "(C)" is correct

$h \propto \cos\theta$ which is not proportional to "θ" $\;\Rightarrow\;$ "D" is incorrect

4.(BD) $i_1 = \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$

$i_2 = \frac{V}{R}\left(1 - e^{-\frac{Rt}{2L}}\right)$



$\Rightarrow \; i = i_1 - i_2 = \frac{V}{R}\left(e^{-\frac{Rt}{2L}} - e^{-\frac{Rt}{L}}\right)$

For i to be max/min

$\frac{di}{dt} = 0 \quad \Rightarrow \quad \frac{di}{dt} = \frac{V}{R}\left(-e^{-\frac{Rt}{2L}}\frac{R}{2L} + e^{-\frac{Rt}{L}}\frac{R}{L}\right) = 0$

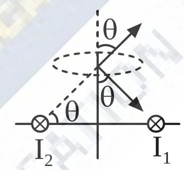
$\Rightarrow \; t = \frac{2L}{R}\ell n(2) = \tau \quad \Rightarrow$ "D" is correct

Also we can see that at $t = \tau$, $\frac{d^2 i}{dt^2}$ is –ve $\Rightarrow$ i is max.

$I_{\max} = \frac{V}{R}\left[\frac{1}{2} - \frac{1}{4}\right] = \frac{V}{4R}$ $\Rightarrow$ "B" is correct

5.(ABD) B due to I_1 at origin $= \frac{\mu_0 I_1}{2\pi R}(+\hat{k})$; B due to I_2 at origin $= \frac{\mu_0 I_2}{2\pi R}(-\hat{k})$

B due to I at origin $= \frac{\mu_0 I}{16R}\left(-\hat{k}\right)$



$\overline{B}_{origin} = \frac{\mu_0}{R}\left(\frac{I_1}{2\pi} - \frac{I_2}{2\pi} - \frac{I}{16}\right)\hat{k}$

So if $I_1 = I_2$ $\Rightarrow$ $B \neq 0$ $\Rightarrow$ "A" is correct

So if $I_1 > I_2$ $\Rightarrow$ I will have a value for which B can be zero. $\Rightarrow$ "B" is correct.

So if $I_1 < 0, I_2 > 0$ $\Rightarrow$ B_{origin} has to be along $-\hat{k}$ $\Rightarrow$ $\Rightarrow$ "C" is incorrect.

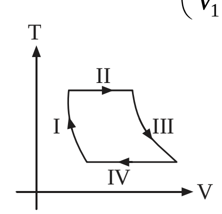
If $I_1 = I_2$ $\Rightarrow$ B at the centre of loop $= \left(-\frac{\mu_0 I}{2R}\right)\hat{k}$ $\Rightarrow$ "D" is Correct.

As Z-component of B due to I_1 & I_2 cancel out

6.(BCD) In process "1", V is changing $\Rightarrow$ "A" is incorrect

In process "2", Temp is constant and volume is increasing $\Rightarrow$ $Q = nRT\,\ell n\left(\frac{V_2}{V_1}\right) = +ve$

$\Rightarrow$ "B" is correct.



Similarly, In process "IV", $Q = -ve$ $\Rightarrow$ "C" is correct

For Isobaric process

$\Rightarrow \; T = \left(\frac{P}{nR}\right)V$

$\Rightarrow$ Straight line passing through origin $\Rightarrow$ "D" is correct

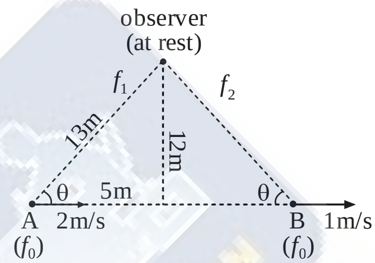
7.(2 or 2.0 or 2.00)

$\overline{A} + \overline{B} = a\left[(1 + \cos\omega t)\hat{i} + \sin\omega t\;\hat{j}\right]$; $\quad \overline{A} - \overline{B} = a\left[(1 - \cos\omega t)\hat{i} - \sin\omega t\;\hat{j}\right]$

$$\left|\overline{A}+\overline{B}\right| = \left|2\cos\left(\frac{\omega t}{2}\right)\right|; \qquad \left|\overline{A}-\overline{B}\right| = \left|2\sin\left(\frac{\omega t}{2}\right)\right|; \qquad \left|\frac{\overline{A}+\overline{B}}{\overline{A}-\overline{B}}\right| = \left|\cot\left(\frac{\omega t}{2}\right)\right| = \sqrt{3}$$

$$\Rightarrow \quad \frac{\omega t}{2} = \frac{\pi}{6} \qquad \Rightarrow \qquad t = 2.$$

8.(5 or 5.0 or 5.00)

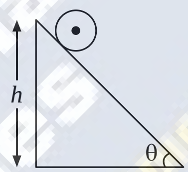


$$f_1 = \frac{f_0\, c}{c - 2\cos\theta}$$

$$f_2 = \frac{f_0\, c}{c + 1\cos\theta}$$

$$\text{Beat frequency } = \left|f_1 - f_2\right| = \frac{f_0\, c\,(3\cos\theta)}{c^2} = \frac{1430\times 3}{330}\times\frac{5}{13} = 5$$

9.(0.75) $t = \sqrt{\dfrac{2(h\,\mathrm{cosec}\,\theta)}{a}}$; $\quad a = \dfrac{g\sin\theta}{\left(1+\dfrac{k^2}{R^2}\right)}$



$$t_{ring} = \sqrt{\frac{2h(1+1)}{g\sin^2\theta}} = \sqrt{h}\,\frac{4}{\sqrt{30}}$$

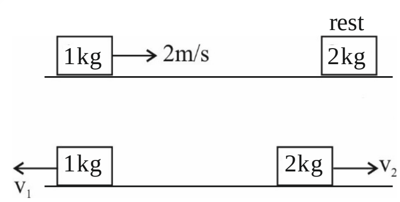
$$t_{Disc} = \sqrt{\frac{2h\left(1+\frac{1}{2}\right)}{g\sin^2\theta}} = \sqrt{h}\,\frac{2\sqrt{3}}{\sqrt{30}}$$

$$\Rightarrow \quad \Delta t = t_{ring} - t_{Disc} = \left(\frac{4-2\sqrt{3}}{\sqrt{30}}\right)\sqrt{h} = \left(\frac{2-\sqrt{3}}{\sqrt{10}}\right)$$

$$\Rightarrow \quad \frac{2}{\sqrt{3}}\sqrt{h} = 1 \qquad \Rightarrow \qquad h = \frac{3}{4} = 0.75$$

10.(2.09) $2v_2 - 1v_1 = 1(2)$ (Momentum conservation)

Also $(v_1 + v_2) = 1(2)$ (Coefficient of restitution)



$$3v_2 = 4 \quad \Rightarrow \quad v_2 = \frac{4}{3} \quad \Rightarrow \quad v_1 = \frac{2}{3}$$

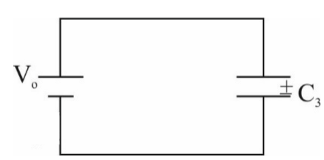
As 2kg is attached to a spring,

Time taken by 2kg to return to its initial position is $t = \dfrac{T}{2} = \pi\sqrt{\dfrac{m}{k}} = \pi\sqrt{\dfrac{2}{2}} = \pi$ sec.

Distance move by 1 kg in this time $v_1 t = \dfrac{2\pi}{3} = 2.093$

11.(1.5 or 1.50)

Initial circuit:



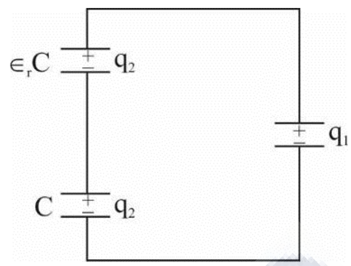
Initial charge on $C_3 = CV_o = 8\mu C$

Final circuit:

By conservation of charge:

$\Rightarrow \quad q_1 + q_2 = 8\mu C$

$\Rightarrow \quad q_2 = 8\mu C - Q_1 = 8\mu C - 5\mu C = 3\mu C$



Also by KVL

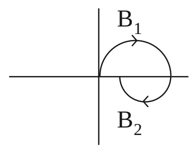
$$\frac{q_2}{\epsilon_r C} + \frac{q_2}{C} = \frac{q_1}{C}$$

$$\Rightarrow \frac{3}{C}\left(\frac{1}{\epsilon_r} + 1\right) = \frac{5}{C} \quad \Rightarrow \epsilon_r = \frac{3}{2} = 1.5$$

12.(2 or 2.0 or 2.00 or 1.2 or 1.20)

$$R_1 = \frac{mv_0}{q\,B_1}, t_1 = \frac{\pi m}{qB_1}$$

$$R_2 = \frac{mv_0}{q\,B_2}, t_2 = \frac{\pi m}{qB_2}$$



Case-1:

Magnitude of average velocity along x-axis

$$= \frac{2R_1 - 2R_2}{t_1 + t_2} = \frac{2\frac{mv_0}{q}\left(\frac{1}{B_1} - \frac{1}{B_2}\right)}{\frac{m\pi}{q}\left(\frac{1}{B_1} + \frac{1}{B_2}\right)} = 2\frac{(\pi)}{\pi}\frac{\left(\frac{1}{B_1} - \frac{1}{4B_1}\right)}{\left(\frac{1}{B_1} + \frac{1}{4B_1}\right)} = 2\left(\frac{3}{5}\right) = 1.2$$

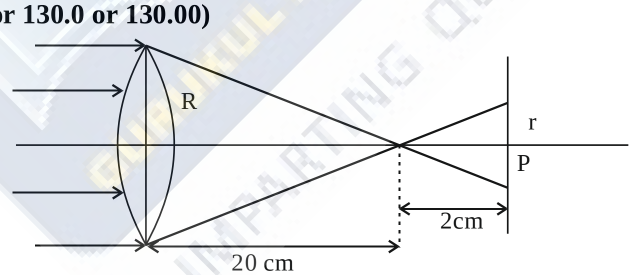
Case-2:

Distance along x-axis = $2R_1 + 2R_2 = 2\frac{mv_0}{q}\left(\frac{1}{B_1} + \frac{1}{B_2}\right)$

Time = $= t_1 + t_2 = \frac{\pi m}{qB_1} + \frac{\pi m}{qB_2}$

Average speed = $\dfrac{\frac{2mv_0}{q}\left(\frac{1}{B_1} + \frac{1}{B_2}\right)}{\frac{\pi m}{q}\left(\frac{1}{B_1} + \frac{1}{B_2}\right)} = \frac{2}{\pi}v_0 = 2$

13.(130 or 130.0 or 130.00)

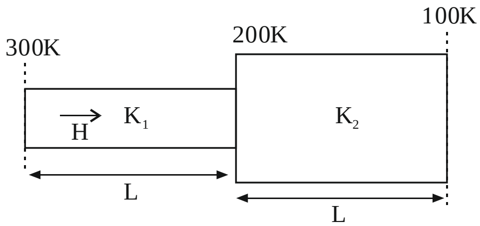


Power incident on the lens = $\mathrm{I} \times \pi R^2$

Intensity at P = $\dfrac{\mathrm{I}(\pi R^2)}{\pi r^2} = \mathrm{I}\left(\frac{R}{r}\right)^2 = \mathrm{I}\left(\frac{20}{2}\right)^2 \quad [\because \frac{R}{r} = \frac{20}{2}$ from similar triangles]

$= 130\ KW/m^2$.

14.(4 or 4.0 or 4.00)



$$H=\frac{K_1\pi(r_1)^2(300-200)}{L}=\frac{K_2\pi(r_2)^2(200-100)}{L} \quad\Rightarrow\quad \frac{K_1}{K_2}=\left(\frac{r_2}{r_1}\right)^2=4$$

15.(C) $v=\frac{E}{B} \quad\Rightarrow\quad [E]=[L][T]^{-1}[B]$

16.(D) $c=\frac{1}{\sqrt{\mu_o \in_o}} \Rightarrow \mu_o=\frac{1}{\in_o (c)^2} \quad\Rightarrow\quad [\mu_0]=[\in_o]^{-1}\ [L]^{-2}\ [T]^2$

17.(B) $r=\frac{(1-a)}{1+a}$

$$\Rightarrow\quad dr=\frac{(1+a)(-da)-(1-a)da}{(1+a)^2}=\frac{2da}{(1+a)^2} \quad\Rightarrow\quad \Delta r=\frac{2\Delta a}{(1+a)^2}$$

18.(C) $N_{decayed}=N_0(1-e^{-\lambda t})$

$$\Rightarrow\quad \lambda=\frac{1}{t}\ln\left(\frac{N_0}{N_0-N_d}\right) \quad\Rightarrow\quad \Delta\lambda=\frac{1}{t}\frac{\Delta N_d}{N_o-N_d}=\frac{40}{2000}=0.02$$

PART-II	CHEMISTRY

1.(BC) $NH_4NO_3 \xrightarrow{\Delta} N_2O + H_2O$

$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + 4H_2O$

$Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2 \uparrow$

$Mg_3N_2 \xrightarrow{T > 300°C}$ decomposition occurs.

2.(BC) Fe $[Ar]3d^6 4s^2$; Ni $[Ar]3d^8 4s^2$: CO is strong field ligand

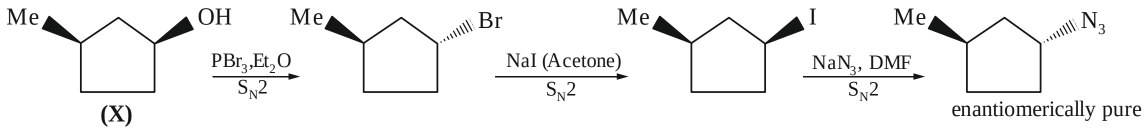
	$Fe(CO)_5$	$Ni(CO)_4$
Valence electrons	$8 + 2 \times 5 = 18$	$10 + 4 \times 2 = 18$
	(Low spin complex)	(Low spin complex)

If oxidation state of M is increased: M – C bond strength decreases and C – O bond strength increases due to less extent of back bonding.
If oxidation state of M is decreased M – C bond strength increases and C – O bond strength decreases due to greater extent of back bonding.

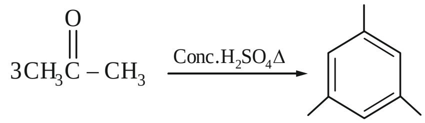
3.(ABC)

A : Metal oxides are more basic in nature	(Correct)
B : NF_3 is more covalent than BiF_3	(Correct)
C : PH_3 boils at lower temperature than NH_3	(Correct)
D : N – N > P – P (bond strength)	(Incorrect)

4.(B)

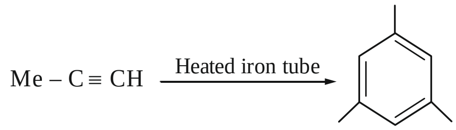


5.ABD **(A)**



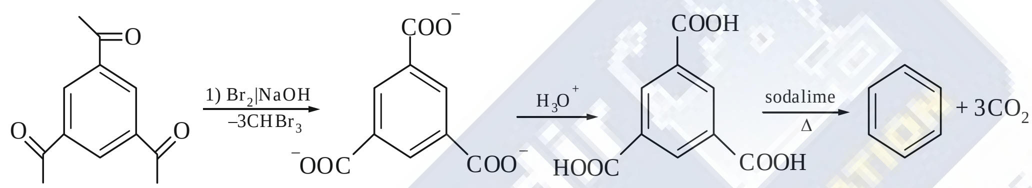
$+ 3H_2O$: Acid catalyzed aldol condensation

(B)

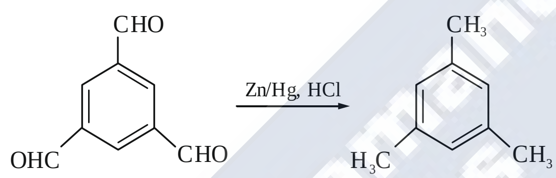


; Gas phase trimerization

(C)

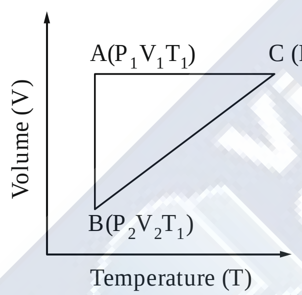


(D)



; Clemmensen's reduction

6.(BC)



Path	AB	BC	CA
Process	Isothermal	Isobaric	Isochoric
	$\Delta U = 0$	$\Delta U = nC_V(T_2 - T_1)$	$\Delta U = nC_V(T_1 - T_2)$
	$\Delta H = 0$	$\Delta H = -nC_p(T_2 - T_1)$	$\Delta H = nC_p(T_1 - T_2)$
	$W = nRT\ln\frac{V_2}{V_1}$	$W = -P_2(V_1 - V_2)$	$W = 0$
	$q = nRTE\ \ln\frac{V_2}{V_1}$	$q = nC_V(T_2 - T_1) + P_2(V_1 - V_2)$	$q = nC_V(T_1 - T_2)$

$q_{AC} = \Delta U_{AC} = nC_V[T_2 - T_1] = \Delta U_{BC}$

$W_{AB} = -nRT_1 \ln\frac{V_2}{V_1}$

$W_{BC} = -P_2[V_1 - V_2] = P_2[V_2 - V_1]$

$q_{BC} = \Delta_{BC} = nC_P[T_2 - T_1] = \Delta H_{AC}$

$\Delta H_{CA} = nC_P[T_1 - T_2]$

$\Delta U_{CA} = nC_V[T_1 - T_2]$

$\Delta H_{CA} < \Delta U_{CA}$ because both are negative and $T_1 < T_2$

7.(1.00)

Paramagnetic	H atom, NO_2 monomer O_2^- (Superoxide),	Odd electron species
	Dimeric sulphur in vapour phase.	According to MOT
	$Mn_3O_4, (NH_4)_2[FeCl_4], (NH_4)_2[NiCl_4], K_2MnO_4$	Unpaired electron in d orbital of central atom
Diamagnetic	K_2CrO_4	No unpaired electron in d orbital of central atom

8.(2992.00)

$$(NH_4)_2SO_4 + Ca(OH)_2 \xrightarrow{100\%} CaSO_4.2H_2O \quad + \quad 2NH_3$$

$$\frac{1584}{132} = 12 \text{ mol} \qquad \frac{1584}{132} = 12 \qquad 2 \times \frac{1584}{132} = 24 \text{ mol}$$

$$NiCl_2 \cdot 6H_2O + 6NH_3 \xrightarrow{100\%} [Ni(NH_3)_6]\,Cl_2 + 6H_2O$$

$$\frac{952}{238} = 4 \text{ mol} \qquad 4 \text{ mol.}$$

Mass of $CaSO_4 \cdot 2H_2O = 12 \times 172 = 2064$ gm

$(M_0 = 172)$

Mass of $[Ni(NH_3)_6]\,Cl_2 = 4 \times 232 = 928$ gm

$(M_0 = 232)$ Ans. = 2992 gm

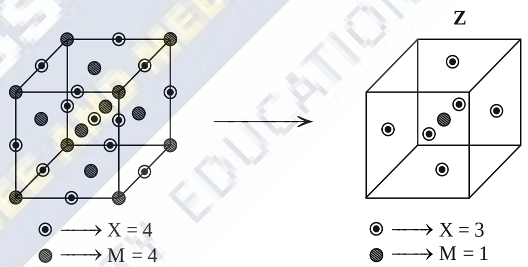
Combined weight (in gm) = 2064 + 228 = 2992

9.(3.00)

M	X
CCP	OHV
4	4

$X \Rightarrow 4 - 3 + 3 - 1 = 3$ (Anion)

$M \Rightarrow 4 - 3 - 1 + 1 = 1$ (Cation)



10.(10.00)

Anode : $Mg \longrightarrow Mg^{2+} + 2e^-$

Cathode: $Cu^{2+} + 2e^- \longrightarrow Cu$

$$Mg + Cu^{2+} \longrightarrow Mg^{2+} + Cu$$

$$E = E^\circ - \frac{RT}{2F}\ell n\frac{[Mg^{2+}]}{[Cu^{2+}]}, \quad 2.70 = E^\circ - \frac{300}{2 \times 11500}\ln\frac{(1)}{(1)}, \quad E = E^\circ = 2.70V$$

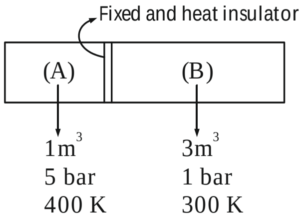
$$2.67 = 2.70 - \frac{300}{2 \times 11500}\ell n\frac{x}{1}$$

$$2.67 - 2.70 = -\frac{300}{2 \times 11500}\ell n\, x$$

$$-0.03 = -0.0130\,\ell n\, x$$

$$2.3 = \ell n\, x \Rightarrow x = 10\,M$$

11.(2.22)



After we changed the partition to movable and conducting, pressure and temperature on both sides will be equal.

Hence $\frac{n_A}{V_A} = \frac{n_B}{V_B}$ ....(1)

Let after sliding volume of chamber A = V

Then volume of chamber $B = 4 - V$

From initial conditions we can estimate n_A and n_B

$$n_A = \frac{P_A V_A}{RT_A} \qquad n_B = \frac{P_B V_B}{RT_B}$$

$$\frac{n_A}{n_B} = \frac{P_A}{P_B} \times \frac{V_A}{V_B} \times \frac{T_B}{T_A} = \frac{(5)}{(1)} \times \frac{(1)}{(3)} \times \frac{(300)}{(400)} = \frac{5}{4}$$

$$\frac{n_A}{n_B} = \frac{5}{4}$$

Using equation:

$$\frac{V_A}{V_B} = \frac{n_A}{n_B}$$

$$\frac{V}{4-V} = \frac{5}{4} \Rightarrow V = \frac{20}{9} = 2.22\ m^3$$

12.(19.00)

Condition I:

$$x_A = 0.5\ ,\ x_B = 0.5$$

$$P_A^0 = 20 \text{ Torr and } P_M = 45 \text{ Torr}$$

Using Raoult's Law

$$P_M = P_A^0 x_A + P_B^0 x_B$$

$$45 = (20)(0.5) + P_B^0(0.5)$$

$$P_B^0 = 70 \text{ Torr}$$

Condition II:

$$x_A = ? \qquad x_B = ?$$

$$P_M = 22.5$$

Using Raoult's Law

$$P_M = P_A^0 x_A + P_B^0 (1 - x_A)$$

$$22.5 = (20)x_A + 70(1 - x_A)$$

$$50x_A = 47.5$$

$$x_A = \frac{47.5}{50} = \frac{19}{20}$$

$$x_A = \frac{19}{20}, \; x_B = \frac{1}{20}$$

$$\frac{x_A}{x_B} = \frac{\frac{19}{20}}{\frac{1}{20}} = 19$$

13.(4.47)

$$AB(S) \rightleftharpoons A^+(aq) + B^-(aq) \quad \ldots\ldots\ldots\ldots\ldots(1)$$

$$B^-(aq) + H^+(aq) \rightleftharpoons HB(aq) \quad \ldots\ldots\ldots\ldots\ldots(2)$$

On adding equation (1) and (2)

$$AB(s) + H^+(aq) \rightleftharpoons A^+(aq) + HB(aq)$$

a	10^{-3}	0	0
$a - s$	10^{-3}	s	s

$$K_{sp} \times \frac{1}{K_a} = \frac{[A^+][HB]}{[H^+]}$$

$$\frac{2 \times 10^{-10}}{1 \times 10^{-8}} = \frac{S^2}{10^{-3}}$$

$$S^2 = 2 \times 10^{-2} \times 10^{-3}$$

$$S^2 = 2 \times 10^{-5}$$

$$S = 4.47 \times 10^{-3} = y \times 10^{-3}$$

$$y = 4.47$$

14.(0.05) **Case : 1**

For solvent X

Solute: NaCl

Molality: m_1

$$i = 2$$

$$\Delta T_b = (362 - 360) = 2 \text{ (from graph)}$$

$$\Delta T_b = i(m)(K_b)_x$$

$$2 = (2)(m)(K_b)_x \ldots\ldots\ldots\ldots..(1) \; ; \; K_{bx} = \frac{1}{m_1}$$

From solvent Y

Solute : NaCl

Molality : m_1

$$i = 2$$

$$\Delta T_b = (368 - 367) = 1 \text{ (from graph)}$$

$$\Delta T_b = i m(K_b)_y$$

$$1 = (2)(m_1)(K_b)_y \ldots\ldots\ldots\ldots..(2) \; ; \; K_{by} = \frac{1}{2m_1}$$

Dividing (1) by (2)

$$\frac{2}{1}=\frac{2\ m_1(K_b)_x}{2\ m_1(K_b)_y}$$

$$\frac{(K_b)_x}{(K_b)_y}=2 \qquad \ldots\ldots(3)$$

Case : 2

For solvent X:

Solute : Non volatile

Molality : m_2

$i=i_x$

$\Delta T_b=3a$

$\Delta T_b=i_x m(K_b)_x$

$3a=i_x m_2(K_b)_x \qquad \ldots\ldots\ldots(4)$

For solvent Y:

Solute : Non volatile

Molality : m_2

$i=i_y$

$\Delta T_b=a$

$\Delta T_b=i_y m(K_b)_y$

$a=i_y m_2(K_b)_y \qquad \ldots\ldots\ldots(5)$

Dividing (4) by (5)

$$\frac{3a}{a}=\frac{i_x}{i_y}\frac{m}{m}\frac{(K_b)_x}{(K_b)_y}$$

$$\frac{i_x}{i_y}=3\times\frac{(K_b)_y}{(K_b)_x}=\frac{3}{2}$$

Now $\quad \frac{i_x}{i_y}=\frac{3}{2}$

In solvent Y:

$2S \longrightarrow S_2$

$1 \qquad 1$

$(1-0.7)\ \ 0.7/2\ \ ;\ i_y=1-\frac{0.7}{2}=\frac{2-0.7}{2}=\frac{1.3}{2}$

In solvent X:

$2S \longrightarrow S_2$

$1 \qquad 1$

$1-\alpha \qquad \frac{\alpha}{2} \quad ;\ i_x=1-\frac{\alpha}{2}$

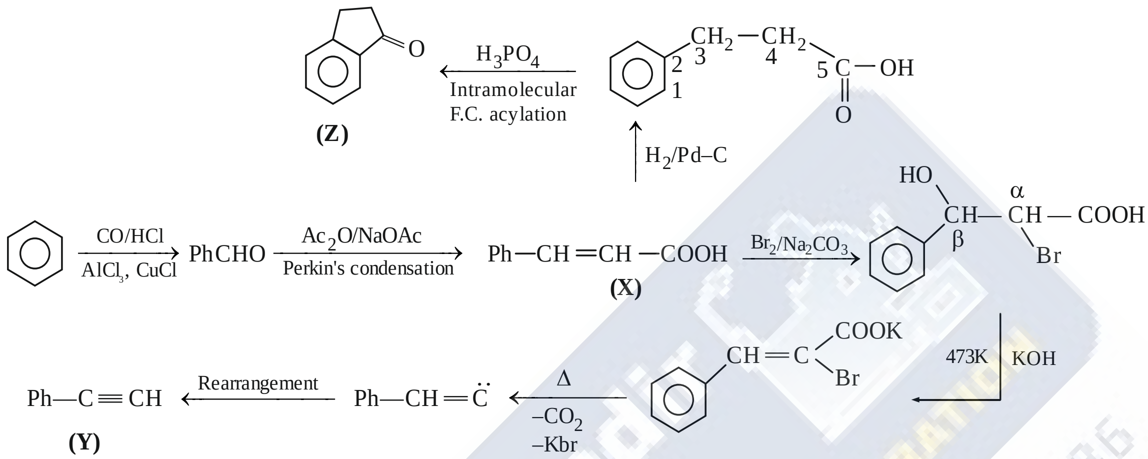
$$\frac{i_x}{i_y}=\frac{3}{2}=\frac{(1-0.5\alpha)}{1.3}\times 2$$

$$\frac{3}{2}=\frac{(1-0.5\alpha)}{1.3}\times 2$$

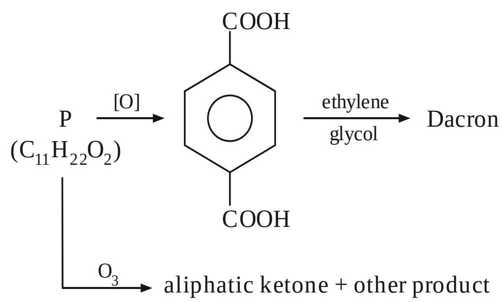
$$\frac{3\times1.3}{4} = (1-0.5\alpha)$$

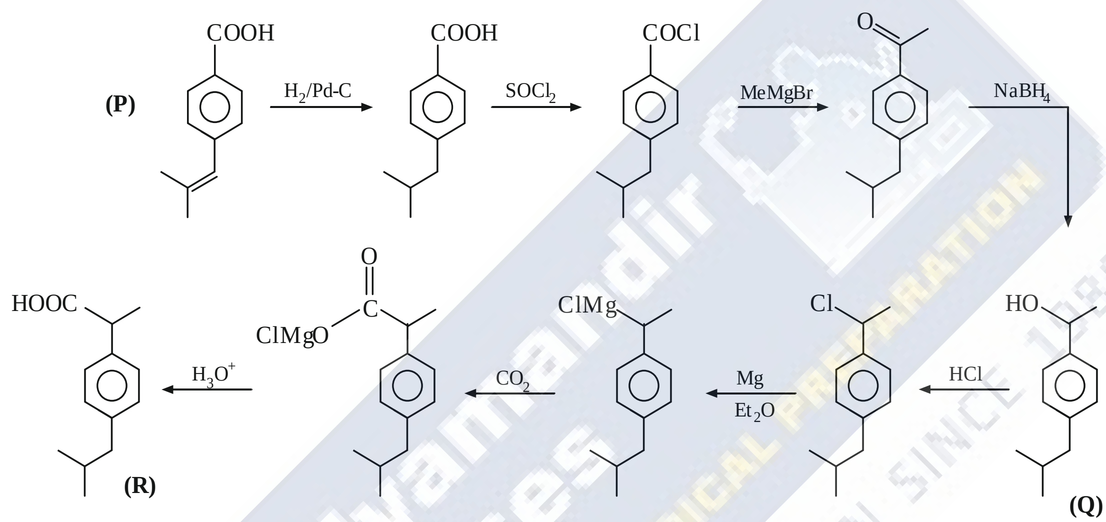
$$\alpha = 0.05$$

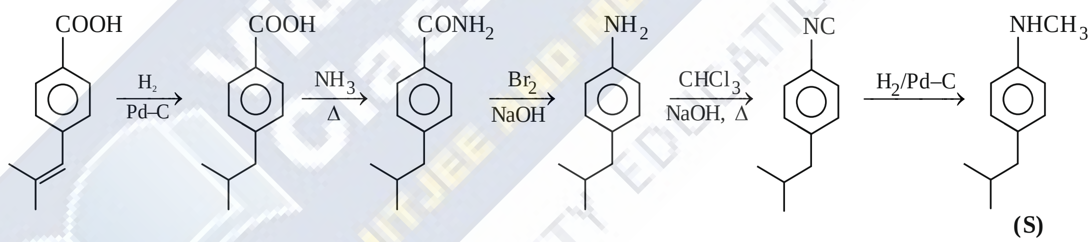
15-16 15.(C) 16.(A)



17-18. 17.(A) 18.(B)







PART-III	MATHEMATICS

1.(ABD) **(i)** $\arg(-1-i) = -\frac{3\pi}{4}$

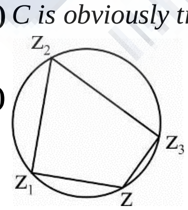
(ii) The function is discontinuous at 0

$$f(0) = \arg(-1) = \pi$$

However $\lim_{t\to 0^-} f(t) = \lim_{t\to 0^-} \arg(-1+2t) = -\pi$

(iii) *C is obviously true*

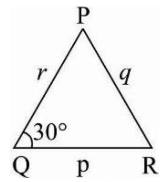
(iv)



In general z lies on a cyclic quadrilateral.

2.(BCD) $r = PQ = 10\sqrt{3};\ QR = 10 = p$

$\cos(30°) = \dfrac{\sqrt{3}}{2} = \dfrac{p^2 + r^2 - q^2}{2pr}$



$\dfrac{\sqrt{3}}{2} = \dfrac{10^2 + 300 - q^2}{2 \cdot 10 \cdot 10\sqrt{3}}$

$q = 10 \quad \Rightarrow \quad \angle RPQ = 30° \quad \text{or} \quad \angle QRP = 120°$

$ar(\Delta PQR) = \dfrac{1}{2} \cdot 10 \cdot 10\sqrt{2} \sin(30°) = 25\sqrt{3}$

$\dfrac{q}{\sin\theta} = 2R \quad \Rightarrow \quad \dfrac{10}{\frac{1}{2}} = 2R \quad \Rightarrow \quad R = 10$

Area of circumcircle $= 100\pi$

$$r = \frac{\Delta}{s} = \frac{25\sqrt{3}}{\frac{1}{2}\left(10+10+10\sqrt{3}\right)} = \frac{25\sqrt{3}}{10+5\sqrt{3}} = \frac{5\sqrt{3}}{2+\sqrt{3}} = 10\sqrt{3} - 15$$

3.(CD) $P_1 : 2x + y - z - 3 = 0; \quad P_2 : x + 2y + z - 2$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} - \hat{i}(1+2) - \hat{j}(2+1) + \hat{k}(4-1) = 3\hat{i} - 3\hat{j} + 3\hat{k}$$

Direction ratios of the line $(1, -1, 1)$

Equation of line $\dfrac{x - 4/3}{3} = \dfrac{y - 1/3}{-3} = \dfrac{z}{3}$

$\left(3\hat{i} - 3\hat{j} + 3\hat{k}\right) \cdot \left(\hat{i} - \hat{j} + \hat{k}\right) \neq 0$

The angle between planes is the angle between their normals.

$$|\cos\theta| = \frac{\left|\left(2\hat{i} + \hat{j} - \hat{k}\right).\left(\hat{i} + 2\hat{j} + \hat{k}\right)\right|}{\sqrt{6}\,\sqrt{6}} = \frac{3}{6} = \frac{1}{2}$$

$\theta = \dfrac{\pi}{3}$

Equation of $P_3 : 1(x-4) - 1(y-2) + 1(z-1) = 0$

$x - y + z - 3 = 0$

Distance from (2, 1, 1) $= \dfrac{|2 - 1 + 1 + 3|}{\sqrt{3}} = \dfrac{2}{\sqrt{3}}$

4.(ABD) Sign preserving property of a continuous function, Say a function f is continuous at $x = a$, then there exists as open interval around a in which f preserves it sign.

W.L.O.G let $f(a) > 0$

Since f is continuous at $x = a$

$\lim\limits_{x \to a} |f(x) - f(a)| < \in \; \forall \in > 0$

Choose $\in = \dfrac{f(a)}{2}$

$-\dfrac{f(a)}{2} < f(x) - f(a) < \dfrac{f(a)}{2}$

$\dfrac{f(a)}{2} < f(x) < \dfrac{3f(a)}{2}$

Therefore there exists some interval in which *f* preserves its sign.

Now consider the interval $(-\delta, \delta)$ where $\delta > 0$. The function can't be constant in the interval as $f'(x) = 0 \ \forall\ x$.

$\Rightarrow \quad f'(0) = 0$ as $\quad \Rightarrow \quad f(0) = \pm\sqrt{85}$ (Not possible)

Which would mean *f* can't be constant around 0.

Now either the function will be one-one in $\left(-\frac{\delta}{2}, \frac{\delta}{2}\right)$ or it won't be. Repeat this process '*n*' times, the interval is $\left(-\frac{\delta}{2^n}, \frac{\delta}{2^n}\right)$. Let '*n*' go to infinity. We arrive upon a contradiction that *f* is constant around 0.

Assume that $|f'(x)| > 1 \forall x \in (-4, 0)$

$f'(x) > 1 \quad$ or $\quad f'(x) < -1$

$\int_{-4}^{0} f'(x)dx > \int_{-4}^{0} 1\, dx \quad$ or $\quad f(-4) < f(0) - 4 < -2 \quad$ or $\quad f(-4) > 4 + f(0) > 2$

Which is absurd.

In fact in any interval of length "*t*".

$|f'(x)|$ can't exceed $\frac{4}{t}$. For all *x*.

Consider $f(x) = 2\sin\left(\frac{\sqrt{85}x}{2}\right)$

$\lim_{x\to\infty} f(x)$ does NOT exist

Consider $g(x) = (f(x))^2 + (f(x))^2$

Say $x_1 < 0 < x_2$

The function is continuous is $[x_1, x_2]$ on account of its is being differentiable $\exists\, \alpha \in (x_1, x_2)$ such that $g'(\alpha) = 0$. (The function will attain an extrema).

As $g(\alpha) \ge g(0) = 85 \Rightarrow g'(\alpha) = 2f'(x)\left(f''(\alpha)\right) + f(\alpha) = 0$

Now $f'(\alpha) \ne 0$ as $f(0) \ge \sqrt{85}$. Which is incorrect.

Therefore $f''(\alpha) + f(\alpha) = 0$.

5.(BC) $e^{-f(x)} f'(x) = e^{-g(x)} g'(x) \ \forall\ x \in R$

$$\int_1^x e^{-f(x)} f'(x)dx = \int_1^x e^{-g(x)}\, g'(x)dx$$

$$\left[-e^{-f(x)}\right]_1^x = \left[-e^{-g(x)}\right]_1^x$$

$$\frac{1}{e} - e^{-f(x)} = e^{-g(1)} - e^{-g(x)}$$

$$e^{-f(x)} = \frac{1}{e} + e^{-g(x)} - e^{-g(1)}$$

$$e^{-f(2)} = \frac{1}{e} + e^{-g(2)} - e^{-g(1)} = \frac{2}{e} - e^{-g(1)} < \frac{2}{e}$$

$$-f(2) < \ln(2) - 1$$

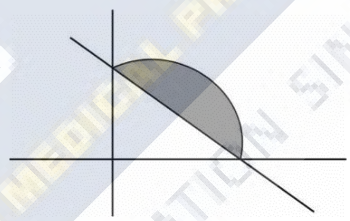
$$f(2) > 1 - \ln 2$$

and $\left[-e^{-f(x)}\right]_2^x = \left[-e^{-g(x)}\right]_2^x$

$$e^{-f(2)} - e^{-f(x)} = e^{-g(2)} - e^{-g(x)}$$
$$e^{-g(x)} = \frac{1}{e} + e^{-f(x)} - e^{-f(2)}$$
$$e^{-g(1)} = \frac{2}{e} - e^{-f(2)} < \frac{2}{e}$$
$$g(1) > 1 - \ln 2$$

6.(BC) $f : [0, \infty) \to R$

$$f(x) = 1 - 2x + \int_0^x e^{x-t} f(t)dt$$
$$f(x) = 1 - 2x + e^x \int_o^x e^{-t}\ f(t)dt$$
$$f(0) = 1\ f'(x) = -2 + \left(e^x(e^{-x}f(x)) + f(x) + 2x - 1\right)$$
$$f'(x) = -2 + 2f(x) + 2x - 1$$
$$f'(x) - 2f(x) = -3 + 2x$$
$$e^{2x}f'(x) - 2e^{2x}f(x) = (-3 + 2x)e^{-2x}$$
$$\frac{d}{dx}\left(e^{-2x}f(x)\right) = (-3 + 2x)e^{-2x}$$
$$\left[e^{-2x}f(x)\right]_0^x = \int_0^x (-3 + 2x)e^{-2x}dx$$
$$f(x) = 1 - x.$$



$$\text{Area} = \frac{\pi}{4}(1)^2 - \frac{1}{2}.1.1$$
$$= \frac{\pi - 2}{4} \text{ sq units}$$

7.(8.00) $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times \left(\sqrt{7}\right)^{\frac{1}{\log_4 7}}$

$$\left((\log_2 9)\right)^{2\times\log^2_{(\log_2 9)}} \times (7)^{\frac{1}{2}\times\log^4_7} = \left((\log_2 9)\right)^{\log^4_{(\log_2 9)}} \times (7)^{\log_7 2} = 4 \times 2 = 8$$

8.(625.00) The last couple of digits can be (12, 24, 32, 44, 52)

Corresponding to every such combination, there are $5 \times 5 \times 5$ such numbers

9.(3748.00) $|X| = 2018$ or $|Y| = 2018$

$T_r = r^{th}$ term of first A.P.

$P_S = s^{th}$ term of second $\Delta \cdot P$

To find $T_r = P_s$ $\quad 1 \le r, s \le 2018$

$$1 + (r-1)5 = 9 + (r-1)7$$

$5r - 7s = 6 \quad \therefore \quad r = 4$ and $s = 2$ is a solution

$\Rightarrow r = 4 - 7t$

$1 \le r \le 2018 \quad \Rightarrow \quad -\frac{2015}{7} \le t \le 2/3$

$\Rightarrow |X \cap Y| = 288 \quad \Rightarrow |X \cup Y| = |X| + |Y| - |X \cap Y| = 3748$

10.(2.00) $\sin^{-1}\left(\sum_{i=1}^{\infty} x^{i+1} - x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^i\right) = \frac{x}{2} - \cos^{-1}\left(\sum_{i=1}^{\infty}\left(-\frac{x}{2}\right) - \sum_{i=1}^{\infty}(-x)^2\right)$

Obviously x = 0 is a solution

$$\sin^{-1}\left(\frac{x^2}{1-x}-\frac{x^2}{2-x}\right)=\sin^{-1}\left(\frac{x}{2+x}-\frac{x}{1+x}\right)$$

$$\frac{x^2}{1-x}-\frac{x^2}{2-x}=-\frac{x}{2+x}+\frac{x}{1+x}$$

$$\frac{x^2+2x-1}{1-x^2}=\frac{x^2+3x-2}{4-x^2}$$

$$4x^2+8x-4-x^4-2x^3+x^2=x^2+3x-2-x^4-3x^3+2x^2$$

$$x^3+2x^2+5x-2=0$$

There exist only one solution in $\left(-\frac{1}{2},\frac{1}{2}\right)$

11.(1.00) $y_n=\frac{1}{n}\left((n+1)(n+2)......(n+n)\right)^{\frac{1}{n}}$

In $y_n=\frac{1}{n}\left(\ln(n+1)(n+2).......(n+n)\right)-\ln(n)$

$$=\frac{1}{n}\sum_{r=1}^{n}\ln(n+r)-\frac{1}{n}\ln(n)^n=\frac{1}{n}\sum_{r=1}^{n}\ln\left(1+\frac{r}{n}\right)$$

$$\lim_{n\to\infty}\ \ln y_n=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\ln\left(1+\frac{r}{n}\right)$$

$$=\int_0^1\ln(1+x)dx=\left[x\left(\ln(1+x)-1\right)+\ln(1+x)\right]_0^1$$

$$\ln\ y_n=\left(2\ln 2-1\right)=\ln 4-1=\ln\frac{4}{e}$$

$$y_n=\frac{4}{e}$$

$$[y_n]=1.=[L]$$

12.(3.00) $\vec{c}=x\vec{a}+y\vec{b}+(\vec{a}\times\vec{b})$

$\vec{a}.\vec{c}=2\cos\alpha=x$

$\vec{b}.\vec{c}=2\cos\alpha=y$

$|\vec{c}|=\sqrt{x^2+y^2+|\vec{a}\times\vec{b}|^2}$

$2=\sqrt{4\cos^2\alpha+4\cos^2\alpha+1}$

$8\cos^2\alpha=3$

13.(1/2) (0.50)

$\sqrt{3}a\cos x+2b\sin x=c,\quad x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\sqrt{3}a\cos\alpha+2b\sin\alpha=c$ .....(i)

$\sqrt{3}a\cos\beta+2b\sin\beta=c$ .....(ii)

(i) - (ii)

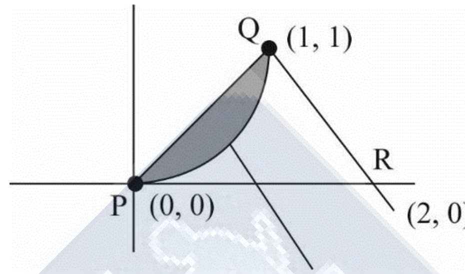
$$-2\sqrt{3}a\sin\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)+2.\ 2b\sin\left(\frac{\alpha-\beta}{2}\right)G\left(\frac{\alpha+\beta}{2}\right)=0$$

$$\Rightarrow\ \tan\left(\frac{\alpha+\beta}{2}\right)=\frac{2b}{\sqrt{3}a}=\frac{2}{3}t\ \text{ where }\ t=\frac{b}{a}$$

$\alpha+\beta=\frac{\pi}{3} \Rightarrow \tan\left(\frac{\alpha+\beta}{2}\right)=\frac{1}{\sqrt{3}}$

$\frac{b}{a}=\frac{1}{2}=(.5)$

14.(4.00) Area of triangle $\Delta PQR=\frac{1}{2}.2.1=1\,\text{sq.units}$



Area taken away by farmer = 0.3 sq. units

$\int_0^1 \left(x-x^n\right)dx=0.3$

$\left[\frac{x^2}{2}\right]_0^1-\left[\frac{x^n}{n+1}\right]_0^1=0.3$

$\frac{1}{2}-\frac{1}{n+1}=\frac{3}{10} \quad \Rightarrow \frac{1}{n+1}=\frac{1}{2}-\frac{3}{10}=\frac{5-3}{10}=\frac{9}{5}$

$\Rightarrow n=4$

15.(A) Clearly $G_3=(2,2)$

$E_3\equiv(0,4)$

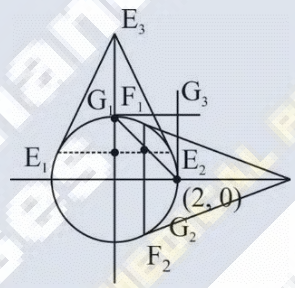
$F_3\equiv(4,0)$

$E_1\equiv\left(-\sqrt{3},\ 1\right)$

$E_2\equiv\left(\sqrt{3},\ 1\right)$

$F_1\equiv\left(1,\ \sqrt{3}\right)$

$F_2\equiv\left(1,\ -\sqrt{3}\right)$



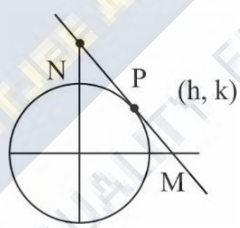
16.(D) Equation of MN

$x\cos\theta+y\sin\theta=2$

$h=\frac{1}{\cos\theta};\ k=\frac{1}{\sin\theta}$

$\left(\frac{1}{h}\right)^2+\left(\frac{1}{k}\right)^2=1$

$\frac{1}{h^2}+\frac{1}{k^2}=1 \Rightarrow x^2+y^2=x^2y^2$



17.(A) $\frac{D_4}{5!}=\frac{4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)}{120}=\frac{3}{40}$

18.(C) Using principle of inclusion and exclusion, the favourable number of cases

$=5!-4.4!.2!+3.3!2!+3.3!2!2!$

$-2.\ 2!2!2!-2.2!2!+2=14$

$P\left(T_1\cap T_2\cap T_3\cap T_4\right)=\frac{14}{120}=\frac{7}{10}$