Read the following Instructions very carefully before you proceed.

GENERAL
1. This sealed booklet is your Question Paper. Do not break the seal till you are told to do so.
2. The paper CODE is printed on the right hand top corner of this sheet and the right hand top corner of the back cover of this booklet.
3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.
4. The paper CODE is printed on the left part as well as the right part of the ORS. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator for change of ORS.
5. Blank spaces are provided within this booklet for rough work.
6. Write your name, roll number and sign in the space provided on the back cover of this booklet.
7. After breaking the seal of the booklet at 2.00 pm, verify that the booklet contains 36 pages and that all the 54 questions along with the options are legible. If not, contact the invigilator for replacement of the booklet.
8. You are allowed to take away the Question Paper at the end of the examination.

OPTICAL RESPONSE SHEET
9. The ORS (top sheet) will be provided with an attached Candidate’s Sheet (bottom sheet). The Candidate’s Sheet is a carbon-less copy of the ORS.
10. Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate’s Sheet.
11. The ORS will be collected by the invigilator at the end of the examination.
12. Your will be allowed to take away the Candidate's Sheet at the end of the examination.
13. Do not tamper with or mutilate the ORS. Do not use the ORS for rough work.
14. Write your name, roll number and code of the examination centre, and sign with pen in the space provided for this purpose on the ORS. Do not write any of these details anywhere else on the ORS. Darken the appropriate bubble under each digit of your roll number.

DARKENING THE BUBBLE ON THE ORS
15. Use a BLACK BALL POINT PEN to darken the bubble on the ORS.
16. Darken the bubble COMPLETELY.
17. The correct way of darkening a bubble is as: ☐
18. The ORS is machine-gradable. Ensure that the bubbles are darkened in the correct way.
19. Darken the bubble ONLY IF you are sure of the answer. There is NO WAY to erase or “un-darken” a darkened bubble.
1. There are two Vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers \( C_1 \) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper \( C_2 \) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values (in cm) by calipers \( C_1 \) and \( C_2 \), respectively, are:

(A) 2.87 and 2.86  (B) 2.85 and 2.82  (C) 2.87 and 2.87  (D) 2.87 and 2.83

1. (D) Reading of vernier scale \( C_1 \) can be calculated by the usual method

Total reading = \( 2.8 \times 10 \text{ mm} + x \)

From figure, \( x = (8 \times 1) \text{ mm} - (7 \times 1.1) \text{ mm} = 0.3 \text{ mm} \)

So, Reading = 2.83 mm

2. The electrostatic energy of \( Z \) protons uniformly distributed throughout a spherical nucleus of radius \( R \) is given by

\[
E = \frac{3}{5} \frac{Z(\text{Z} - 1)e^2}{4\pi\varepsilon_0 R}
\]

The measured masses of the neutron, \( \frac{1}{2}H \), \( \frac{15}{7}N \) and \( \frac{15}{8}O \) are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u, respectively. Given that the radii of both the \( \frac{15}{7}N \) and \( \frac{15}{8}O \) nuclei are same, 1u = 931.5 MeV/c\(^2\) (\( c \) is the speed of light) and \( \varepsilon^2/(4\pi\varepsilon_0) = 1.44 \text{ MeV fm} \). Assuming that the difference between the binding energies of \( \frac{15}{7}N \) and \( \frac{15}{8}O \) is purely due to the electrostatic energy, the radius of either of the nuclei is:

(A) 2.85 fm  (B) 3.03 fm  (C) 3.42 fm  (D) 3.80 fm

2. (C) \( E_N = \frac{3}{5} \frac{7 \times 6e^2}{4\pi\varepsilon_0 R} = \frac{126}{5} \frac{e^2}{4\pi\varepsilon_0 R} \)
\[ E_0 = \frac{3}{5} \frac{8 \times 7e^2}{4\pi \varepsilon_0 R} = \frac{156}{5} \frac{e^2}{4\pi \varepsilon_0 R} \]

\[ \Delta E = E_0 - E_N = \frac{42}{5} \frac{e^2}{4\pi \varepsilon_0 R} \]

\[ \text{Binding energy of } ^{15}_7 N : \quad E_1 = 7 \times 1.007825 + 8 \times 1.008665 - 15.000109 \]

\[ \text{Binding energy of } ^{15}_8 O : \quad E_2 = 8 \times 1.007825 + 7 \times 1.008665 - 15.003065 \]

\[ \text{Difference in B. E.} = |E_1 - E_2| = -1.007825 + 1.008665 + 0.002956 = 0.003796 \text{ u} \]

This is equal to \( \Delta E \) in equation (i)

\[ R = 3.42 \text{ fm} \]

3. The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1 m at 10\(^\circ\)C. Now the end P is maintained at 10\(^\circ\)C, while the end S is heated and maintained at 400\(^\circ\)C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is \( 1.2 \times 10^{-5} \text{ K}^{-1} \), the change in length of the wire PQ is:

(A) 0.78 mm  \quad (B) 0.90 mm  \quad (C) 1.56 mm  \quad (D) 2.34 mm

3(A)

Since thermal resistance of RS is twice that of PQ, temperature of junction QR will be 140\(^\circ\)C. Variation of temperature from P to Q will be linear. Hence, \[ \Delta l = l \propto [T - T_0] \]

Temperature of center of PQ can be taken as T. i.e. \( \Delta l = 1 \times 1.2 \times 10^{-5} [75 - 10] = 1.2 \times 65 \times 10^{-5} = 0.78 \text{ mm} \]

Hence (A)

4. A small object is placed 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle \( \theta = 30^\circ \) to the axis of the lens, as shown in the figure.

If the origin of the coordinate system is taken to be at the centre of the lens, the coordinates (in cm) of the point \((x, y)\) at which the image is formed are:

(A) \((25, 25\sqrt{3})\) \quad (B) \((0, 0)\) \quad (C) \((125/3, 25\sqrt{3})\) \quad (D) \((50 - 25\sqrt{3}, 25)\)
4. (A) Image formed by lens is at (75, 0).
This acts as a virtual image for mirror.
Suppose that mirror is not tilted
Then, for mirror
\[ u = +25, V = ?, F = +50 \]
\[ \frac{1}{v} + \frac{1}{25} = \frac{1}{50} \]
\[ \Rightarrow \quad v = -50 \text{ cm} \quad \cdots \cdots \cdot (i) \]
This image is formed on x-axis
Now, if the mirror is rotated clockwise by 30º, image rotates by 60º clockwise. Look at the ray striking the pole of mirror. This ray rotates by 60º. Image lies on this ray. Hence image rotates by 60º.
New co-ordinates of image will be
\[ x = 50 - 50 \cos 60^\circ = 25 \text{ cm} \]
\[ y = 50 \sin 60^\circ = \frac{50 \sqrt{3}}{2} \text{ cm} \]
Hence (A)

5. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure \( P_i = 10^5 \text{ Pa} \) and volume \( V_i = 10^{-3} \text{ m}^3 \) changes to a final state at \( P_f = (1/32) \times 10^5 \text{ Pa} \) and \( V_f = 8 \times 10^{-3} \text{ m}^3 \) in an adiabatic quasi-static process, such that \( P^\gamma V^\delta \) = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at \( P_i \) followed by an isochoric (isovolumetric) process at volume \( V_f \). The amount of heat supplied to the system in the two-step process is approximately:

(A) 112 J  \quad (B) 294 J  \quad (C) 588 J  \quad (D) 813 J

5. (C) In the first process:
\[ P_i V_i^\gamma = P_f V_f^\gamma \]
\[ \Rightarrow \quad \frac{P_i}{P_f} = \left( \frac{V_f}{V_i} \right)^\gamma \quad \Rightarrow \quad 32 = 8^\gamma \]
\[ \Rightarrow \quad \gamma = \frac{5}{3} \quad \cdots \cdots \cdot (i) \]
For the two step process
\[ W = P_i (V_f - V_i) = 10^5 (7 \times 10^{-3}) \]
\[ W = 7 \times 10^2 \text{ J} \]
\[ \Delta U = \frac{\lambda}{2} (P_i V_f - P_i V_i) = \frac{1}{\lambda - 1} \left( \frac{1}{4} \times 10^2 - 10^2 \right) \]
\[ \Delta U = -\frac{3}{2} \times 10^2 = -\frac{9}{8} \times 10^2 \text{ J} \]
\[ Q - W = \Delta U \quad \Rightarrow \quad Q = 7 \times 10^2 - \frac{9}{8} \times 10^2 = \frac{47}{8} \times 10^2 \text{ J} = 588 \text{ J} \]

6. An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use?

(A) 64  \quad (B) 90  \quad (C) 108  \quad (D) 120
6. (C) \[
\frac{dN}{dt} = \lambda N = 64(8) \text{ where } X \text{ is the permissible limit of Radioactivity}
\]

Laboratory can be considered safe when \[
\frac{dN}{dt} = X = \lambda N_0 e^{-\lambda t}
\]

\[
\Rightarrow \frac{\lambda N_0}{64} = N_0 e^{-\lambda t} \Rightarrow e^{-\lambda t} = \frac{1}{64} \Rightarrow \lambda t = \ln(64) \Rightarrow \frac{\ln(2)}{t_{1/2}}(t) = 6\ln(2)
\]

\[
\Rightarrow t = 6t_{1/2} = 108 \text{ days}
\]

SECTION II (Maximum Marks : 32)

- This section contains EIGHT questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:
  - Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
  - Partial Marks : +1 For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened.
  - Zero Marks : 0 If none of the bubbles is darkened.
  - Negative Marks : -2 In all other cases.
- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in -2 marks, as a wrong option is also darkened.

7. Two thin circular discs of mass \(m\) and \(4m\), having radii of \(a\) and \(2a\), respectively, are rigidly fixed by a massless, rigid rod of length \(l = \sqrt{3}a\) through their centres. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is \(\omega\). The angular momentum of the entire assembly about the point ‘O’ is \(L\) (see the figure). Which of the following statement(s) is (are) true?

(A) The magnitude of angular momentum of the assembly about its center of mass is \(17ma^2\omega/2\)
(B) The center of mass of the assembly rotates about the \(z\)-axis with an angular speed of \(\omega/5\)
(C) The magnitude of the \(z\)-component of \(\vec{L}\) is \(55ma^2\omega\)
(D) The magnitude of angular momentum of center of mass of the assembly about the point O is \(81ma^2\omega\)

7.(AB)
For (A)

For Angular momentum about CM, we will calculate \( L \) about axis marked as (1)

\[
L_1 = \frac{ma^2}{2} \omega + \frac{4m(2a)^2}{2} \omega
\]

\[
= \frac{17ma^2}{2} \omega \Rightarrow A \text{ is correct}
\]

For (B)

\[
V_{CM} = \frac{m(\omega a) + 4m(\omega 2a)}{5m} = \frac{9\omega a}{5}
\]

\[
l_{CM} = \frac{m(l) + 4m(2l)}{5m} = \frac{9l}{5}
\]

\[
\Omega_{about(2)} = \frac{V_{CM}}{l_{CM}} = \frac{\omega a}{l}
\]

Component of \( \Omega \) along z-axis = \( \Omega \cos \theta = \frac{\omega a}{l} \frac{l}{5} \frac{\omega}{5} \Rightarrow B \text{ is correct}

For (C)

Moment of inertia about axis marked as (2)

\[
I_2 = \left( \frac{ma^2}{4} + ml^2 \right) + \frac{4m(2a)^2}{4} + 4m(2l)^2 = \frac{17ma^2}{4} + 17ml^2
\]

So,

\[
L_2 = I_2 \Omega \text{ and net } L_{about \ z-axis} = L_2 \cos \theta - L_4 \sin \theta = 80.75ma^2 \omega \Rightarrow C \text{ is incorrect}
\]

For (D),

\[
L_{CM \ about \ O} = 5m(V_{CM}) \left( \frac{9l}{5} \right) = 5m \frac{9\omega a}{5} \frac{9l}{5} = 81 m\omega a^2 \sqrt{24} \Rightarrow D \text{ is incorrect}
\]

8. A rigid wire loop of square shape having side of length \( L \) and resistance \( R \) is moving along the x-axis with a constant velocity \( v_0 \) in the plane of the paper. At \( t = 0 \), the right edge of the loop enters a region of length 3L where there is a uniform magnetic field \( B_0 \) into the plane of the paper, as shown in the figure. For sufficiently large \( v_0 \), the loop eventually crosses the region. Let \( x \) be the location of the right edge of the loop. Let \( v(x) \), \( I(x) \) and \( F(x) \) represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of \( x \). Counter-clockwise current is taken as positive.

Which of the following schematic plot(s) is(are) correct? (Ignore gravity)
8. (BD) At a general position \( x \),
\[
i = \frac{BLV}{R}
\]
\[
F = -\frac{B^2L^2V}{R}
\]
\[
\therefore \quad \frac{mv}{dx} = -\frac{B^2L^2}{R} \cdot V \quad \Rightarrow \quad V = V_0 - \frac{B^2L^2x}{mR}
\]
i.e. velocity will decrease linearly.

When the loop is fully within field, force and current will be zero and velocity is constant. While coming out, current will be clockwise, force backward and velocity will decrease linearly. Hence, (B), (D)

9. In an experiment to determine the acceleration due to gravity \( g \), the formula used for the time period of a periodic motion is 
\[
T = 2\pi \sqrt{\frac{7(R-r)}{5g}}
\]
The values of \( R \) and \( r \) are measured to be \((60 \pm 1) \) mm and \((10 \pm 1) \) mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.57 s, 0.54 s and 0.59 s. The least count of the watch used for the measurement of time period is 0.01 s. Which of the following statement(s) is(are) true?

(A) The error in the measurement of \( r \) is 10%  
(B) The error in the measurement of \( T \) is 3.57%  
(C) The error in the measurement of \( T \) is 2%  
(D) The error in the determined value of \( g \) is 11%

9. (ABD) Mean time period 
\[
= \frac{0.52 + 0.56 + 0.57 + 0.54 + 0.59}{5} = 0.556 \pm 0.56 \text{sec as per significant figures}
\]
Error in reading 
\[
= |T_{\text{mean}} - T_1| = 0.04
\]
\[
|T_{\text{mean}} - T_2| = 0.00  
|T_{\text{mean}} - T_3| = 0.01
\]
\[
|T_{\text{mean}} - T_4| = 0.02
\]
\[
|T_{\text{mean}} - T_5| = 0.03
\]
Mean error 
\[
= 0.1/5 = 0.02
\]
% error in \( T \) 
\[
= \frac{\Delta T}{T} \times 100 = \frac{0.02}{0.56} \times 100 = 3.57%
\]
% error in \( r \) 
\[
= \frac{0.001 \times 100}{0.010} = 10%
\]
% error in \( R \) 
\[
= \frac{0.001 \times 100}{0.060} = 1.67%
\]
% error in \( \frac{\Delta g}{g} \times 100 = \frac{\Delta(R-r)}{R-r} \times 100 + 2 \times \frac{\Delta T}{T} = \frac{0.002 \times 100}{0.05} + 2 \times 3.57 = 4\% + 7\% = 11\%
\]

10. Light of wavelength \( \lambda_{ph} \) falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is \( \phi \) and the anode is a wire mesh of conducting material kept at a distance \( d \) from the cathode. A potential difference \( V \) is maintained between the electrodes. If the minimum de Broglie wavelength of the electrons passing through the anode is \( \lambda_e \), which of the following statement(s) is(are) true?

(A) For large potential difference \( V >> \phi / e \), \( \lambda_e \) is approximately halved if \( V \) is made four times

(B) \( \lambda_e \) decreases with increase in \( \phi \) and \( \lambda_{ph} \)

(C) \( \lambda_e \) increases at the same rate as \( \lambda_{ph} \) for \( \lambda_{ph} < \frac{hc}{\phi} \)
(D) $\lambda_e$ is approximately halved, if $d$ is doubled

10. (A) $K.E_{\text{max}}$ of $e^-$ just after ejection

$$K.E_i = \frac{hc}{\lambda_{ph}}$$

$$K.E_{\text{max}} e^- \text{ reaching anode } K.E_f = \left( \frac{hc}{\lambda_{ph}} - \phi \right) + eV$$

For $V \gg \frac{\phi}{e} \Rightarrow K.E_f \approx eV \Rightarrow \lambda e = \frac{h}{\sqrt{2m(eV)}} \propto \frac{1}{\sqrt{V}}$

$\Rightarrow$ (A) is correct

If $\phi$ & $\lambda_{ph}$ increase

$\Rightarrow K.E_f$ decreases $\Rightarrow \lambda e$ increases

$\Rightarrow$ (B) is incorrect

$\frac{d\lambda e}{dt} \neq \frac{d\lambda_{ph}}{dt} \Rightarrow$ (C) is incorrect

$\lambda_e$ is independent of $d$ $\Rightarrow$ (D) is incorrect

11. Consider two identical galvanometers and two identical resistors with resistance $R$. If the internal resistance of the galvanometers $R_C < R / 2$, which of the following statement(s) about any one of the galvanometers is(are) true?

(A) The maximum voltage range is obtained when all the components are connected in series
(B) The maximum voltage range is obtained when the two resistors and one galvanometer are connected in series, and the second galvanometer is connected in parallel to the first galvanometer
(C) The maximum current range is obtained when all the components are connected in parallel
(D) The maximum current range is obtained when the two galvanometers are connected in series and the combination is connected in parallel with both the resistors

11. (BC)

For (A),

$$V_1 = 2i_g R + 2i_g R_C$$

For (B),

$$R_C = V_1 + i_g \left( 2R - R_C \right)$$

$\Rightarrow V_2 > V_1$ as $\frac{R}{2} > R_C$, Hence $2R > R_C$

$\Rightarrow$ B is correct

For (C),

$$I_1 = 2i_g + 2i = 2i_g + 2i \frac{R_C}{R}$$
12. While conducting the Young’s double slit experiment a student replaced the two slits with a large opaque plate in the x-y plane containing two small holes that act as two coherent point sources \((S_1, S_2)\) emitting light of wavelength 600 nm. The student mistakenly placed the screen parallel to the x-z plane (for \(z > 0\)) at a distance \(D = 3\) m from the mid-point of \(S_1S_2\), as shown schematically in the figure. The distance between the sources \(d = 0.6003\) mm. The origin \(O\) is at the intersection of the screen and the line joining \(S_1S_2\). Which of the following is(are) true of the intensity pattern on the screen?

(A) Hyperbolic bright and dark bands with foci symmetrically placed about \(O\) in the \(x\)-direction

(B) Straight bright and dark bands parallel to the \(x\)-axis

(C) Semi circular bright and dark bands centered at point \(O\)

(D) The region very close to the point \(O\) will be dark

12. (CD) At any point on screen, which is at a distance \(r\) from \(O\), in the plane of the screen path difference will depend only on \(r\).

\[ \Rightarrow \] all such points have same path difference.

\[ \Rightarrow \] bands will have circular shape.

At \(O\), path difference = \(d = 0.6003\) mm

\[ = 100.5 \lambda = \frac{(2n+1)\lambda}{2} \]

\[ \Rightarrow \] Dark band at \(O\)

13. A block with mass \(M\) is connected by a massless spring with stiffness constant \(k\) to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude \(A\) about an equilibrium position \(x_0\). Consider two cases: (i) when the block is at \(x_0\); and (ii) when the block is at \(x = x_0 + A\). In both the cases, a particle with mass \(m(<M)\) is softly placed on the block after which they stick to each other. Which of the following statement(s) is(are) true about the motion after the mass \(m\) is placed on the mass \(M\)?

(A) The amplitude of oscillation in the first case changes by a factor of \(\frac{M}{M+m}\), whereas in the second case it remains unchanged

(B) The final time period of oscillation in both the cases is same

(C) The total energy decreases in both the cases

(D) The instantaneous speed at \(x_0\) of the combined masses decreases in both the cases
13. (ABD) \[ \omega' = \sqrt{\frac{k}{M + m}} \quad \omega_0 = \sqrt{\frac{k}{M}} \]

When \( m \) placed at mean position,

Let velocity of M & m be \( v' \) just after placing the block

By momentum conservation,

\[ M v_0 = (M + m)v' \quad \Rightarrow \quad M \omega A = (M + m) \omega' A' \]

\[ \Rightarrow \quad \frac{A'}{A} = \frac{M}{(M + m)} \omega = \sqrt{\frac{M}{M + m}} \]

When \( m \) placed at extreme position,

\[ v_M \text{ before placing} = 0 \quad \Rightarrow \quad v_{M+m} \text{ after placing} = 0 \]

\[ \Rightarrow \quad \text{extreme position and mean positions remain unchanged} \]

\[ \Rightarrow \quad A' = A \]

\[ T' = \frac{2\pi}{\omega}, \text{ which is same in both cases.} \]

Energy decreases in 1st case, but not in 2nd case.

Velocity at mean position = \( \omega' A' \) which decreases in both cases.

14. In the circuit shown below, the key is pressed at time \( t = 0 \).

Which of the following statement(s) is (are) true?

(A) The voltmeter displays –5 V as soon as the key is pressed, and displays +5 V after a long time.

(B) The voltmeter will display 0 V at time \( t = \ln 2 \) seconds.

(C) The current in the ammeter becomes \( \frac{1}{e} \) of the initial value after 1 second.

(D) The current in the ammeter becomes zero after a long time.

14. (ABCD)

Just after pressing key,

\[ 5 - 25000i_1 = 0 \]

\[ 5 - 50000i_2 = 0 \]

(As charge in both cap = 0)

\[ \Rightarrow \quad i_1 = 0.2 \ mA \quad \Rightarrow \quad i_2 = 0.1 \ mA \]

And \( V_B + 25000i_1 = V_A \quad \Rightarrow \quad V_B - V_A = -5V \)

After a long time, \( i_1 \) & \( i_2 \) = 0 (steady state)

\[ \Rightarrow \quad 5 - \frac{q_1}{40} = 0 \quad \Rightarrow \quad q_1 = 200\mu C \]

And \( 5 - \frac{q_2}{20} = 0 \quad \Rightarrow \quad q_2 = 100\mu C \]

\[ V_B - \frac{q_2}{20} = V_A \quad \Rightarrow \quad V_B - V_A = +5V \quad \Rightarrow \quad \text{(A) is correct.} \]
For capacitor 1, \( q_1 = 200 \left[ 1 - e^{-t/1} \right] \mu C \)

\[ i_1 = \frac{1}{5} e^{-t/1} \text{mA} \]

For capacitor 2, \( q_2 = 100 \left[ 1 - e^{-t/1} \right] \mu C \)

\[ i_2 = \frac{1}{10} e^{-t/1} \text{mA} \]

\[ V_B - \frac{q_2}{20} + i_1 \times 25 = V_A \]

\[ \Rightarrow \quad V_B - V_A = 5 \left[ 1 - e^{-t} \right] - 5e^{-t} = 5 \left[ 1 - 2e^{-t} \right] \]

At \( t = \frac{\pi}{2} \), \( V_B - V_A = 5[1 - 1] = 0 \)

\( \Rightarrow \quad \text{(B) is correct.} \)

At \( t = 1 \), \( i = i_1 + i_2 = \frac{1}{5} e^{-1} + \frac{1}{10} e^{-1} = \frac{3}{10} \cdot \frac{1}{e} \)

At \( t = 0 \), \( i = i_1 + i_2 = \frac{1}{5} \cdot \frac{1}{10} = \frac{3}{10} \)

\( \Rightarrow \quad \text{(C) is correct.} \)

After a long time, \( i_1 = i_2 = 0 \quad \Rightarrow \quad \text{(D) is correct.} \)

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**SECTION III (Maximum Marks : 12)**

- This section contains **TWO** paragraph.
- Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:
  - **Full Marks** : +3    If only the bubble corresponding to the correct answer is darkened.
  - **Zero Marks** : 0    In all other cases.

**Paragraph for Questions 15 - 16**

A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity \( \omega \) is an example of a non-inertial frame of reference. The relationship between the force \( \vec{F}_{rot} \) experienced by a particle of mass \( m \) moving on the rotating disc and the force \( \vec{F}_{in} \) experienced by the particle in an inertial frame of reference is

\[ \vec{F}_{rot} = \vec{F}_{in} + 2m \left( \vec{v}_{rot} \times \vec{\omega} \right) + m \left( \vec{\omega} \times \vec{r} \right) \times \vec{\omega} \]

Where \( \vec{v}_{rot} \) is the velocity of the particle in the rotating frame of reference and \( \vec{r} \) is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius \( R \) rotating counter-clockwise with a constant angular speed \( \omega \) about its vertical axis through its center. We assign a coordinate system with the origin at the center of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the
z-axis along the rotation axis \( \vec{\omega} = \omega \hat{k} \). A small block of mass \( m \) is gently
placed in the slot at \( r = \left( R/2 \right) \hat{i} \) at \( t = 0 \) and is constrained to move only along
the slot.

15. The distance \( r \) of the block at time \( t \) is:

(A) \( \frac{R}{4} \left( e^{2\omega t} + e^{-2\omega t} \right) \)

(B) \( \frac{R}{4} \left( e^{2\omega t} + e^{-2\omega t} \right) \)

(C) \( \frac{R}{2} \cos 2\omega t \)

(D) \( \frac{R}{2} \cos 2\omega t \)

15. (B) In the expression \( \vec{F}_{\text{rot}} = \vec{F}_{\text{in}} + 2m(\vec{V}_{\text{rot}} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega} \)

The third term is radially outward i.e. this goes towards increasing \( \vec{V}_{\text{rot}} \).

2nd term is perpendicular to edge of slot and is cancelled by Normal reaction from edge of slot.

So, for radial direction:

\[
F = m\omega^2 r
\]

\[
\frac{md^2r}{dt^2} = m\omega^2 r
\]

i.e.

\[
\frac{d^2r}{dt^2} = \omega^2 r
\]

Solution to this differential equation is \( r = A e^{\omega t} + B e^{-\omega t} \)

Where A and B are constants

* for detailed solution of above differential equation see the end of this solution .

At \( t = 0 \), \( r = \frac{R}{2} \)

\( \frac{R}{2} = A + B \)

Also, \( V_{\text{rot}} = \frac{dr}{dt} = A\omega e^{\omega t} - B\omega e^{-\omega t} \)

At \( t = 0 \), \( V_{\text{rot}} = 0 \), so,

\[ 0 = (A - B)\omega \]

\[ \therefore A = B = \frac{R}{4} \]

So,

\[ r = \frac{R}{4} \left[ e^{\omega t} + e^{-\omega t} \right] \]

Hence (B)

* Detailed solution of differential equation

\[
\Rightarrow a = \omega^2 r = v \frac{dv}{dr}
\]

\[
\Rightarrow \int v \ dv = \omega^2 \int \frac{r \ dr}{R/2}
\]

\[
\Rightarrow \frac{v^2}{2} = \omega^2 \left[ \frac{r^2}{2} - \frac{R^2}{8} \right]
\]

\[
\Rightarrow \int \frac{dr}{R/2 \sqrt{r^2 - R^2/4}} = \int \omega \ dt
\]

\[
\Rightarrow \ln \left[ \frac{r + \sqrt{r^2 - R^2/4}}{R/2} \right] = \omega t
\]

\[
\Rightarrow r^2 - \frac{R^2}{4} = \left[ \frac{R}{2} e^{\omega t} - R \right]^2
\]

\[
\Rightarrow r^2 - \frac{R^2}{4} = \left[ \frac{R}{2} e^{\omega t} + r^2 - R \right] e^{\omega t} \]
\[ r \frac{R}{4} e^{\omega t} = \frac{R^2}{4} \left[ 1 + e^{2\omega t} \right] \quad \Rightarrow \quad r = \frac{R}{4} \left[ e^{\omega t} + e^{-\omega t} \right] \]

16. The net reaction of the disc on the block is:

(A) \(-m\omega^2 R \cos \omega t \hat{j} - mg \hat{k}\)

(B) \(\frac{1}{2} m\omega^2 R \left( e^{2\omega t} - e^{-2\omega t} \right) \hat{j} + mg \hat{k}\)

(C) \(m\omega^2 R \sin \omega t \hat{j} - mg \hat{k}\)

(D) \(\frac{1}{2} m\omega^2 R \left( e^{\omega t} - e^{-\omega t} \right) \hat{j} + mg \hat{k}\)

16.(D) From previous question, term \(2m \left( \vec{V}_{\text{rot}} \times \vec{\omega} \right)\)
gives normal reaction from edge \(N_e\)

\[ V_{\text{rot}} = \frac{dr}{dt} = \frac{R}{4} \left[ e^{\omega t} - e^{-\omega t} \right] \omega = \frac{R\omega}{4} \left( e^{\omega t} - e^{-\omega t} \right) \]

So,

\[ N_e = 2m \frac{R\omega}{4} \left( e^{\omega t} - e^{-\omega t} \right) \omega \]

\[ \therefore \quad \vec{N}_e = \frac{M\omega R}{2} \left( e^{\omega t} - e^{-\omega t} \right) \hat{j} \]

Normal reaction from bottom of slot = \(mg \hat{k}\)

\[ \therefore \quad \text{Net reaction from slot} \quad \vec{R} = \frac{M\omega R}{2} \left( e^{\omega t} - e^{-\omega t} \right) \hat{j} + mg \hat{k} \]

Hence (D)

**Paragraph for Questions 17 - 18**

Consider an evacuated cylindrical chamber of height \(h\) having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius \(r << h\). Now a high voltage source \((HV)\) is connected across the conducting plates such that the bottom plate is at \(+V_0\) and the top plate at \(-V_0\).

Due to their conducting surface the balls will get charged, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. (Ignore gravity)

17. Which one of the following statements is correct?

(A) The balls will bounce back to the bottom plate carrying the opposite charge they went up with

(B) The balls will stick to the top plate and remain there

(C) The balls will execute simple harmonic motion between the two plates

(D) The balls will bounce back to the bottom plate carrying the same charge they went up with

17. (A) Balls will gain + ve charge and hence move towards – ve plate.
On reaching –ve plate, balls will attain –ve charge and come back to +ve plate. So on, balls will keep oscillating. But oscillation is not S.H.M., As fore on balls is not $\propto x$.
⇒ (A) is correct.

18. The average current in the steady state registered by the ammeter in the circuit will be:
(A) proportional to $\sqrt{V_0}$
(B) zero
(C) proportional to $V_0^2$
(D) proportional to the potential $V_0$

18.(C) As the balls keep on carrying charge from one plate to another, current will keep on flowing even in steady state. When at bottom plate, if all balls attain charge $q$,

$$\frac{kq}{r} = V_0$$

$$\Rightarrow q = \frac{V_0 r}{k}$$

Inside cylinder, electric field $E = \left[V_0 - (-V_0)\right]h = 2V_0h$.

$$\Rightarrow$$ Acceleration of each ball, $a = \frac{qE}{m} = \frac{2hr}{km}V_0^2$

$$\Rightarrow$$ Time taken by balls to reach other plate, $t = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2h km}{2hrV_0^2}} = \frac{1}{V_0} \sqrt{\frac{km}{r}}$

If there are $n$ balls,

Average current, $i_{av} = \frac{nq}{t} = n \times \frac{V_0 r}{k} \times \sqrt{\frac{r}{km}} \Rightarrow i_{av} \propto V_0^2$

SECTION I (Maximum Marks : 18)

- This section contains SIX questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:
  - Full Marks: +3 If, only the bubble corresponding to the correct option is darkened.
  - Zero Marks: 0 If none of the bubbles is darkened.
  - Negative Marks: -1 In all other cases.

19. The correct order of acidity for the following compounds is

| (A) | I > II > III > IV | (B) | III > I > II > IV | (C) | III > IV > II > I | (D) | I > III > IV > II |

19.(A) According to ortho effect ortho substituted benzoic acid is strongest acid among o, m and p substituted benzoic acid. +M effect of OH decreases acidic strength of benzoic acid. The correct order of acidity is I > II > III > IV.
20. The major product of the following reaction sequence is

\[ \text{O} \quad \text{i) HCHO (excess)/NaOH, heat} \]
\[ \text{HCHO/H}^+, \text{(catalytic amount)} \]

\[ \text{(A)} \quad \text{(B)} \quad \text{(C)} \quad \text{(D)} \]

20.(A) \[ \begin{align*}
&\text{phenyl} \\
&\xrightarrow{\text{HCHO}} \\
&\text{phenyl} \\
&\xrightarrow{\text{HCHO/H}^+, \text{catalytic amount}} \text{CH}_2\text{OH}
\end{align*} \]

21. In the following reaction sequence in aqueous solution, the species X, Y and Z, respectively, are:

\[ \text{S}_2\text{O}_3^{2-} \xrightarrow{\text{Ag}^+} X \quad \text{clear solution} \quad \text{Ag}^+ \quad \text{Y white precipitate} \quad \text{with time} \quad \text{Z black precipitate} \]

\[ \text{(A)} \quad [\text{Ag(S}_2\text{O}_3)_2]^{3-}, \text{Ag}_2\text{S}_2\text{O}_3, \text{Ag}_2\text{S} \]
\[ \text{(B)} \quad [\text{Ag(S}_2\text{O}_3)_3]^{3-}, \text{Ag}_2\text{SO}_3, \text{Ag}_2\text{S} \]
\[ \text{(C)} \quad [\text{Ag(SO}_3)_2]^{3-}, \text{Ag}_2\text{S}_2\text{O}_3, \text{Ag} \]
\[ \text{(D)} \quad [\text{Ag(SO}_3)_3]^{3-}, \text{Ag}_2\text{SO}_4, \text{Ag} \]

21.(A) \[ \begin{align*}
&\text{S}_2\text{O}_3^{2-} \xrightarrow{\text{Ag}^+} [\text{Ag(S}_2\text{O}_3)_2]^{3-} \xrightarrow{\text{Ag}^+} \text{Ag}_2\text{S}_2\text{O}_3(s) \xrightarrow{\text{Z black precipitate}} \text{Ag}_2\text{S}(s)
\end{align*} \]

22. The qualitative sketches I, II and III given below show the variation of surface tension with molar concentration of three different aqueous solutions of KCl, CH$_3$OH and CH$_3$(CH$_2$)$_{11}$OSO$_3$Na$^+$ at room temperature. The correct assignment of the sketches is:

\[ \text{Surface tension} \quad \text{Concentration} \]
\[ \text{I} : \text{KCl} \quad \text{II} : \text{CH}_3\text{OH} \quad \text{III} : \text{CH}_3(CH_2)_{11}\text{OSO}_3\text{Na}^+ \]
\[ \text{Surface tension} \quad \text{Concentration} \]
\[ \text{I} : \text{CH}_3(CH_2)_{11}\text{OSO}_3\text{Na}^+ \quad \text{II} : \text{CH}_3\text{OH} \quad \text{III} : \text{KCl} \]
\[ \text{Surface tension} \quad \text{Concentration} \]
\[ \text{I} : \text{CH}_3\text{OH} \quad \text{II} : \text{CH}_3(CH_2)_{11}\text{OSO}_3\text{Na}^+ \quad \text{III} : \text{CH}_3\text{OH} \]
(D)  I : CH₃OH  II : KCl  III : CH₃(CH₂)₁₁OSO₃Na⁺

22. (D)

![Surface Tension Graph]

For KCl curve - Increase of surface tension for inorganic salts
For CH₃OH curve - Decrease of surface tension progressively for alcohols
For CH₃(CH₂)₁₁OSO₃Na⁺ curve - Decrease of surface tension before CMC (Critical Micelle Concentration) and then almost unchanged

23. The geometries of the ammonia complexes of Ni²⁺, Pt²⁺ and Zn²⁺, respectively, are :
   (A) octahedral, square planar and tetrahedral  (B) square planar, octahedral and tetrahedral
   (C) tetrahedral, square planar and octahedral  (D) octahedral, tetrahedral and square planar

23.(A) [Ni(NH₃)₆]²⁺ ⇒ sp³d² ⇒ Octahedral   [Pt(NH₃)₄]²⁺ ⇒ dsp² ⇒ Square planar
   [Zn(NH₃)₄]²⁺ ⇒ sp³ ⇒ Tetrahedral

24. For the following electrochemical cell at 298 K,

\[
\begin{align*}
\text{Anode} & : \text{H}_2 & \rightarrow & 2\text{H}^+ + 2\text{e}^- \\
\text{Cathode} & : \text{M}^{4+} + 2\text{e}^- & \rightarrow & \text{M}^{2+} \\
\text{H}_2 + \text{M}^{4+} & \rightarrow & 2\text{H}^+ + \text{M}^{2+} \\
E_{\text{cell}} & = 0.092 \text{ V} \quad \text{when} \quad \frac{\text{M}^{2+}(aq)}{\text{M}^{4+}(aq)} = 10^x \quad \text{Given:} \quad E_{\text{M}^{4+}/\text{M}^{2+}}^0 = 0.151 \text{ V} \quad 2.303 \frac{RT}{F} = 0.059 \text{ V}
\end{align*}
\]

The value of \(x\) is :
   (A) –2  (B) –1  (C) 1  (D) 2

24.(D) \(E_{\text{cell}}^0 = E_{\text{c}}^0 - E_{\text{a}}^0\)

\[
\begin{align*}
\text{Anode} & : \text{H}_2 & \rightarrow & 2\text{H}^+ + 2\text{e}^- \\
\text{Cathode} & : \text{M}^{4+} + 2\text{e}^- & \rightarrow & \text{M}^{2+} \\
\text{H}_2 + \text{M}^{4+} & \rightarrow & 2\text{H}^+ + \text{M}^{2+} \\
E_{\text{cell}}^0 & = 0.151 - 0 = 0.151
\end{align*}
\]

\[
E_{\text{cell}} = E_{\text{cell}}^0 - 0.059 - \log \left( \frac{\text{M}^{2+}(aq)}{\text{M}^{4+}(aq)} \right)_{\text{pH}_2}
\]

\[
0.092 = 0.151 - 0.059 - \log \left( \frac{10^x \times 1}{1} \right)
\]

\[
0.059 = \frac{0.059}{2} \log 10^x
\]

\[
\log 10^x = 2
\]

\[
x = 2
\]
This section contains **EIGHT** questions.

Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.

For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.

For each question, marks will be awarded in one of the following categories:

- **Full Marks**: +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened.
- **Partial Marks**: +1 For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened.
- **Zero Marks** : 0 If none of the bubbles is darkened.
- **Negative Marks**: –2 In all other cases.

For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in –2 marks, as a wrong option is also darkened.

25. According to Molecular Orbital Theory,
(A) \( \text{C}_2^2^- \) is expected to be diamagnetic
(B) \( \text{O}_2^2^+ \) is expected to have a longer bond length than \( \text{O}_2 \)
(C) \( \text{N}_2^+ \) and \( \text{N}_2^- \) have the same bond order
(D) \( \text{He}_2^2^+ \) has the same energy as two isolated He atoms

25. (AC) (A) is true because all electrons are paired.
(B) is false \( B.O. \text{O}_2 = 2, \ B.O. \text{O}_2^2^+ = 6 - 0 = 3 \)
(Bond length) \( \text{O}_2^2^- < \) (Bond length) \( \text{O}_2^+ \)
(C) Both have B.O. = 2.5 (True)
(D) No it will have lesser energy. (False)

26. The **CORRECT** statement(s) for cubic close packed (ccp) three dimensional structure is(are) :
(A) The number of the nearest neighbours of an atom present in the topmost layer is 12
(B) The efficiency of atom packing is 74%
(C) The number of octahedral and tetrahedral voids per atom are 1 and 2, respectively.
(D) The unit cell edge length is \( 2\sqrt{2} \) times the radius of the atom

26. (BCD) CCP has a packing efficiency of 74%.
The nearest neighbours in the top most layer is 9.
The number of tetrahedral and octahedral voids are 2 and 1 respectively per atom.
The edge length is \( 2\sqrt{2} \) times the radius of atom \( (\sqrt{2}a = 4r) \)

27. Reagent(s) which can be used to bring about the following transformation is(are) :
(A) LiAlH\(_4\) in \((\text{C}_2\text{H}_5)_2\text{O}\)
(B) BH\(_3\) in THF
(C) NaBH\(_4\) in \(\text{C}_2\text{H}_5\text{OH}\)
(D) Raney Ni/H\(_2\) in THF

27. (C) LiAlH\(_4\) in Et\(_2\)O, BH\(_3\) in THF & Raney Ni/H\(_2\) in THF reduces aldehydic group, carboxylic acid and ester groups. NaBH\(_4\) in EtOH reduces only aldehyde groups.

28. Extraction of copper from copper pyrite (CuFeS\(_2\)) involves
(A) crushing followed by concentration of the ore by froth-flotation
(B) removal of iron as slag
(C) self-reduction step to produce ’blist’er copper’ following evolution of SO$_2$
(D) refining of ‘blist’er copper’ by carbon reduction

28. (ABC)
CuFeS$_2$

(A) The ore contains sulphide ions so in the purification, concentration is done by froth-floatation process.
(B) $2\text{CuFeS}_2 + \text{O}_2 \rightarrow \text{Cu}_2\text{S} + 2\text{FeS} + \text{SO}_2$

$2\text{FeS} + 3\text{O}_2 \rightarrow 2\text{FeO} + 2\text{SO}_2$

$\text{FeO} + \text{SiO}_2 \rightarrow \text{FeSiO}_3$

(slag)

(C) $2\text{Cu}_2\text{S} + 3\text{O}_2 \rightarrow 2\text{Cu}_2\text{O} + 2\text{SO}_2$ (Partial oxidation)

$2\text{Cu}_2\text{O} + \text{Cu}_2\text{S} \rightarrow 6\text{Cu} + \text{SO}_2$ (Self Reduction)

(D) Copper is refined by electrolytic refining

29. The nitrogen containing compound produced in the reaction of HNO$_3$ with P$_2$O$_5$

(A) can also be prepared by reaction of P$_4$ and HNO$_3$
(B) is diamagnetic
(C) contains one N-N bond
(D) reacts with Na metal producing a brown gas

29. (BD) 1. $\text{P}_4 + 2\text{HNO}_3 \rightarrow 4\text{H}_2\text{PO}_4 + 20\text{NO}_2 + 4\text{H}_2\text{O}$

Hence, N$_2$O$_5$ cannot be prepared using P$_4$ and HNO$_3$

2. $\text{P}_2\text{O}_{10} + 2\text{HNO}_3 + 5\text{H}_2\text{O} \rightarrow \text{N}_2\text{O}_5 + 4\text{H}_3\text{PO}_4$

3. N$_2$O$_5$ is formed which is diamagnetic and does not have N–N bond.

4. N$_2$O$_5$ + Na $\rightarrow$ NaNO$_3$ + NO$_2$ (↑)

Brown coloured gas

30. Mixture(s) showing positive deviation from Raoult’s law at 35°C is(are):

(A) carbon tetrachloride + methanol
(B) carbon disulphide + acetone
(C) benzene + toluene
(D) phenol + aniline

30. (AB) Positive deviation is observed when the interaction between A and B in mixture is weaker than the interaction between A and B alone.

(Interaction between A–A + interaction between B–B) > (2×interaction between A–B).

Benzene and toluene forms an ideal solution while phenol and aniline form a solution showing –ve deviation from Raoult’s law.

31. For ‘invert sugar’, the correct statement(s) is(are):

(Given: specific rotations of (+) –sucrose, (+) –maltose, L–(-)-glucose and L–(+)-fructose in aqueous solution are +66º, +140º, –52º and +92º, respectively)

(A) ‘invert sugar’ is prepared by acid catalyzed hydrolysis of maltose
(B) ‘invert sugar’ is an equimolar mixture of D–(+)-glucose and D–(–)-fructose
(C) specific rotation of ‘invert sugar’ is –20º
(D) on reaction with Br$_2$ water, ‘invert sugar’ forms saccharic acid as one of the products

31. (BC) Maltose$\xrightarrow{\text{H}_2\text{O}}$ 2D–(–)-glucose

Sucrose$\xrightarrow{\text{H}_2\text{O}}$ D–(–)-glucose + L–(–)-fructose

Invert sugar

Specific rotation of invert sugar is –20º (Observed rotation is +52º – 92º = –40º, which is for 2 moles of mixture). On reaction with Br$_2$ water, invert sugar forms gluconic acid as one of the product [HOOC(CHOH)$_4$CH$_2$OH]

32. Among the following, reaction(s) which gives(give) tert-butyll benzene as the major product is(are):
SECTION III (Maximum Marks : 12)

- This section contains TWO questions.
- Based on each paragraph, there are TWO questions.
- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:
  - Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.
  - Zero Marks : 0 In all other cases.

Paragraph for Questions 33 - 34

Thermal decomposition of gaseous $X_2$ to gaseous $X$ at 298 K takes place according to the following equation:

$$X_2(g) \xrightleftharpoons{298 K} 2X(g)$$

The standard reaction Gibbs energy, $\Delta_r G^\circ$, of this reaction is positive. At the start of the reaction, there is one mole of $X_2$ and no $X$. As the reaction proceeds, the number of moles of $X$ formed is given by $\beta$. Thus, $\beta_{equilibrium}$ is the number of moles of $X$ formed at equilibrium. The reaction is carried out at a constant total pressure of 2 bar. Consider the gases to behave ideally. (Given: $R = 0.083$ L bar K$^{-1}$ mol$^{-1}$)

33. The equilibrium constant $K_P$ for this reaction at 298 K, in terms of $\beta_{equilibrium}$, is:

(A) $\frac{8\beta^2_{equilibrium}}{2 - \beta_{equilibrium}}$  
(B) $\frac{8\beta^2_{equilibrium}}{4 - \beta^2_{equilibrium}}$

(C) $\frac{4\beta^2_{equilibrium}}{2 - \beta_{equilibrium}}$  
(D) $\frac{4\beta^2_{equilibrium}}{4 - \beta^2_{equilibrium}}$

33. (B)
34. The INCORRECT statement among the following, for this reaction, is
(A) Decrease in the total pressure will result in formation of more moles of gaseous X
(B) At the start of the reaction, dissociation of gaseous X₂ takes place spontaneously
(C) \( \beta_{\text{equilibrium}} = 0.7 \)
(D) \( K_c < 1 \)

34.(C) (A) It is true statement \( X_2(g) \rightleftharpoons 2X(g) \)

If \( P \) is decreased then reaction will move in forward direction according to Le-Chatelier's principle.

(B) It is true statement
At start of reaction \( Q = 0 \)
\[ \therefore \Delta G = \Delta G^0 + RT \ln Q \]
If \( Q = 0 \)
Then \( \Delta G = -\text{ve}, \therefore \text{Reaction is spontaneous} \)

(D) It is true statement
\[ \Delta G^0 = +\text{ve} \]
\[ -RT \ln K_p = \Delta G^0 \]
\[ \Rightarrow K_p < 1 \text{ since } K_c < K_p \Rightarrow K_c \text{ is also less than 1.} \]

Paragraph for Questions 35 - 36

Treatment of compound \( O \) with \( \text{KMnO}_4/\text{H}^+ \) gave \( P \), which on heating with ammonia gave \( Q \). The compound \( Q \) on treatment with \( \text{Br}_2/\text{NaOH} \) produced \( R \). On strong heating, \( Q \) gave \( S \), which on further treatment with ethyl 2-bromopropanoate in the presence of \( \text{KOH} \) followed by acidification, gave a compound \( T \).

35. The compound \( R \) is:
35. (A)

36. The compound T is:
(A) glycine  (B) alanine  (C) valine  (D) serine

36. (B)

37. Let \( P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} \) and I be the identity matrix of order 3. If \( Q = \begin{bmatrix} q_{11} \\ \vdots \\ q_{33} \end{bmatrix} \) is a matrix such that \( P^{50} - Q = I \), then \( \frac{q_{31} + q_{32}}{q_{21}} \) equals:
(A) \( \frac{52}{52} \)  (B) \( 103 \)  (C) \( 201 \)  (D) \( 205 \)

37. (B) \( P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} \) ; \( P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 8 & 1 & 0 \\ 48 & 8 & 1 \end{bmatrix} \) ; \( P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 96 & 12 & 1 \end{bmatrix} \) ; \( P^4 = \begin{bmatrix} 1 & 0 & 0 \\ 16 & 1 & 0 \\ 160 & 16 & 1 \end{bmatrix} \)

Pattern of element \( b_{31} \) is 16[1,3,6,10,.....]
\[ 50^{th} \text{ term is } 16 \times 1275 \]
[By observing that \( T_n \) of \( S = 1 + 3 + 6 + 10 + \ldots \) is \( \frac{n^2 + n}{2} \)]

\[
p^{S_0} = \begin{bmatrix} 1 & 0 & 0 \\ 200 & 1 & 0 \\ 16 \times 1275 & 200 & 1 \end{bmatrix}; \quad Q = p^{S_0} - I = \begin{bmatrix} 0 & 0 & 0 \\ 200 & 0 & 0 \\ 16 \times 1275 & 200 & 0 \end{bmatrix}
\]

\[
\therefore \frac{q_{31} + q_{32}}{q_{21}} = 16 \times 1275 + 200 = 102 + 1 = 103
\]

38. The value of \( \sum_{k=1}^{13} \sin \left( \frac{\pi}{4} + \frac{(k-1)\pi}{6} \right) \sin \left( \frac{\pi}{4} + \frac{k\pi}{6} \right) \) is equal to:

(A) \( 3 - \sqrt{3} \) \quad (B) \( 2(3 - \sqrt{3}) \) \quad (C) \( 2(\sqrt{3} - 1) \) \quad (D) \( 2(2 + \sqrt{3}) \)

38.(C) \[
2 \sum_{k=1}^{13} \sin \left( \frac{\pi}{4} + \frac{(k-1)\pi}{6} \right) \sin \left( \frac{\pi}{4} + \frac{k\pi}{6} \right) = 2 \sum_{k=1}^{13} \left[ \cot \left( \frac{\pi}{4} + \frac{(k-1)\pi}{6} \right) - \cot \left( \frac{\pi}{4} + \frac{k\pi}{6} \right) \right]
\]

\[
= 2 \left[ \cot \left( \frac{\pi}{4} \right) - \cot \left( \frac{\pi}{4} + \frac{13\pi}{6} \right) \right] = 2 \left[ 1 - \cot \left( \frac{\pi}{4} + \frac{\pi}{6} \right) \right] = 2 \left( 1 - \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \right)
\]

\[
= 2 \left[ 1 - \frac{4 - 2\sqrt{3}}{2} \right] = 2 \left[ 1 - 2 + \sqrt{3} \right] = 2\left( \sqrt{3} - 1 \right)
\]

39. Let \( b_i > 1 \) for \( i = 1, 2, \ldots, 101 \). Suppose \( \log_e b_1, \log_e b_2, \ldots, \log_e b_{101} \) are in Arithmetic Progression (A.P.) with the common difference \( \log_e 2 \). Suppose \( a_1, a_2, \ldots, a_{101} \) are in A.P. such that \( a_1 = b_1 \) and \( a_{51} = b_{51} \). If \( t = b_1 + b_2 + \ldots + b_{51} \) and \( s = a_1 + a_2 + \ldots + a_{51} \), then

(A) \( s > t \) and \( a_{101} > b_{101} \) \quad (B) \( s > t \) and \( a_{101} < b_{101} \)

(B) \( s < t \) and \( a_{101} > b_{101} \) \quad (D) \( s < t \) and \( a_{101} < b_{101} \)

39.(B) \[
\log_e \left( \frac{b_2}{b_1} \right) = \log_e (2) \Rightarrow b_2 = 2b_1
\]

\[
\therefore \ b_1, b_2, \ldots, a_{101} \text{ are in G.P. with common ratio, } r = 2.
\]

Let \( a_1 = b_1 = y \quad y > 1 \) \quad (given)

and \( a_{51} = b_{51} = x \)

\[
s = \frac{51}{2} (a_1 + a_{51}) = \frac{51}{2} (x + y)
\]

\[
t = b_1 [2^{51} - 1] = 2^{51} \cdot b_1 = 2^{51} \cdot y - y
\]

\[
s - t = \frac{51}{2} x + \frac{51}{2} y = 2^{51} \cdot y + y
\]

Also \( b_{51} = b_1 \cdot 2^{50} \Rightarrow x = y \cdot 2^{50} \)

\[
s - t = \frac{51}{2} x + \frac{53}{2} y = 2 \cdot x = \frac{47}{2} x + \frac{53}{2} y > 0 \quad \therefore \quad s > t
\]

Also \( a_{101} = 2a_{51} - a_1 \Rightarrow a_{101} = 2x - y
\]

\[
b_{101} = \left( \frac{b_{51}}{b_1} \right)^2 = \frac{x^2}{y}
\]
\[ b_{101} - a_{101} = \frac{x^2}{y} - 2x + y = \frac{x^2 - 2xy + y^2}{y} = \frac{(x - y)^2}{y} > 0 \]

\( \therefore b_{101} > a_{101} \)

40. The value of \( \int_{-\pi/2}^{\pi/2} \frac{x^2 \cos x}{1 + e^x} \, dx \) is equal to:

(A) \( \frac{\pi^2}{4} - 2 \) \quad (B) \( \frac{\pi^2}{4} + 2 \) \quad (C) \( \pi^2 - e^\frac{\pi}{2} \) \quad (D) \( \pi^2 + e^\frac{\pi}{2} \)

40.(A) 
\[ I = \int_{-\pi/2}^{\pi/2} \frac{x^2 \cos x}{1 + e^x} \, dx \]
\[ \Rightarrow I = \int_0^{\pi/2} x^2 \cos x \, dx \left( 1 + \frac{1}{1 + e^x} + \frac{1}{1 + e^{-x}} \right) \, dx \]
\[ \Rightarrow I = \int_0^{\pi/2} x^2 \cos x \, dx = x^2 \sin x \bigg|_0^{\pi/2} - 2 \int_0^{\pi/2} x \sin x \, dx \]
\[ = \frac{\pi^2}{4} - 2 \left[ x \cos x \bigg|_{\pi/2}^0 + \int_0^{\pi/2} \cos x \, dx \right] = \frac{\pi^2}{4} - 2[1] = \frac{\pi^2}{4} - 2 \]

41. Let \( P \) be the image of the point \( (3, 1, 7) \) with respect to the plane \( x - y + z = 3 \). Then the equation of the plane passing through \( P \) and containing the straight line \( \frac{x}{1} = \frac{y}{2} = \frac{z}{1} \) is:

(A) \( x + y - 3z = 0 \) \quad (B) \( 3x + z = 0 \) \quad (C) \( x - 4y + 7z = 0 \) \quad (D) \( 2x - y = 0 \)

41.(C) Let image be \( P(h, k, \ell) \)
\[ \frac{h - 3}{1} = \frac{k - 1}{-1} = \frac{\ell - 7}{1} = -\frac{2}{3}[6] = (-4) \]
\[ h = -1 \quad k = 5 \quad \ell = 3 \]
\( \therefore P = (-1, 5, 3) \)

Other point \( = (0, 0, 0) \)
\[ \therefore \quad \left| \begin{array}{ccc} i & j & k \\ 1 & -5 & -3 \\ 1 & 2 & 1 \end{array} \right| = i - 4j + 7k \]
\( \therefore \quad \text{Plane is } x - 4y + 7z = 0 \)

42. The area of the region \( \{(x, y) \in R^2 : y \geq \sqrt{x+3}, 5y \leq x + 9 \leq 15\} \) is equal to:

(A) \( \frac{1}{6} \) \quad (B) \( \frac{4}{3} \) \quad (C) \( \frac{3}{2} \) \quad (D) \( \frac{5}{3} \)

42.(C) Shaded area = Area of trapezium – Area of \( AED \) – Area of \( EBC \)
43. Let $P$ be the point on the parabola $y^2 = 4x$ which is at the shortest distance from the center $S$ of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$. Let $Q$ be the point on the circle dividing the line segment $SP$ internally. Then:

(A) $SP = 2\sqrt{5}$

(B) $SO : OP = (\sqrt{5} + 1) : 2$

(C) the $x$-intercept of the normal to the parabola at $P$ is 6

(D) the slope of the tangent to the circle at $Q$ is $\frac{1}{2}$

43. (ACD) $S(2, 8), r = \sqrt{4 + 64 - 64} = 2$

\[
\frac{2t - 8}{t^2 - 2} = -t
\]

\[
2t - 8 = -t^3 + 2t
\]

$t = 2$

$P(4, 4)$

$PS = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$

$SQ = 2$

$SO : OP = 2 : (2\sqrt{5} - 2) = 1 : \sqrt{5} - 1 = (\sqrt{5} + 1) : 4$

Normal at point $2x + y = 12$

$x$-intercept = 6
Slope of tangent at Q is \( \frac{1}{2} \).

44. Let \( \hat{u} = u_1 \hat{i} + u_2 \hat{j} + u_3 \hat{k} \) be a unit vector in \( \mathbb{R}^3 \) and \( \hat{w} = \frac{1}{\sqrt{6}}(\hat{i} + \hat{j} + 2\hat{k}) \). Given that there exists a vector \( \vec{v} \) in \( \mathbb{R}^3 \) such that \( |\hat{u} \times \vec{v}| = 1 \) and \( \hat{w} \cdot (\hat{u} \times \vec{v}) = 1 \). Which of the following statement(s) is (are) correct?

(A) There is exactly one choice for such \( \vec{v} \)

(B) There are infinitely many choices for such \( \vec{v} \)

(C) If \( \hat{u} \) lies in the xy-plane then \( |u_1| = |u_2| \)

(D) If \( \hat{u} \) lies in the xz-plane then \( 2|u_1| = |u_3| \)

44.(BC) \( \hat{w} \cdot (\hat{u} \times \vec{v}) = 1 \) \( \Rightarrow \) \( |\hat{w}| |\hat{u} \times \vec{v}| \cdot \cos \theta = 1 \)

\( \Rightarrow \) \( \cos \theta = 1 \) \( \Rightarrow \) \( 0 = 0 \)

\( \Rightarrow \) \( \hat{w} \) is \( \perp \) to plane of \( \hat{u} \) & \( \vec{v} \)

\( \Rightarrow \) \( \hat{w} \) is \( \perp \) \( \hat{u} \) and \( \hat{w} \) is \( \perp \) \( \vec{v} \)

\( \Rightarrow \) \( \hat{w} \) is \( \perp \) to \( \hat{u} \) \( \Rightarrow \) \( u_1 + u_2 + 2u_3 = 0 \)

If \( \hat{u} \) in xy plane \( \Rightarrow \) \( u_3 = 0 \)

\( \Rightarrow \) \( u_1 = -u_2 \) \( \Rightarrow \) \( |u_1| = |u_2| \)

If \( \hat{u} \) in xz plane \( \Rightarrow \) \( u_2 = 0 \)

\( \Rightarrow \) \( u_1 + 2u_3 = 0 \) \( \Rightarrow \) \( |u_1| = 2 |u_3| \)

For \( \hat{v}, \hat{w} \) \( \perp \) \( \vec{v} \) and \( |\hat{u} \times \vec{v}| = 1 \) \( \Rightarrow \) \( |\vec{v}| \sin \alpha = 1 \)

\( \alpha \) is angle b/w \( \hat{u} \) and \( \vec{v} \)

So \( \vec{v} \) can take many values where \( |\vec{v}| > 1 \) and \( \hat{w} \cdot \vec{v} = 0 \)

45. Let \( a, b \in \mathbb{R} \) and \( f : \mathbb{R} \to \mathbb{R} \) be defined by \( f(x) = a \cos(|x^3 - x|) + b |x| \sin(|x^3 + x|) \). Then \( f \) is

(A) differentiable at \( x = 0 \) if \( a = 0 \) and \( b = 1 \)

(B) differentiable at \( x = 1 \) if \( a = 1 \) and \( b = 0 \)

(C) NOT differentiable at \( x = 0 \) if \( a = 1 \) and \( b = 0 \)

(D) NOT differentiable at \( x = 1 \) if \( a = 1 \) and \( b = 1 \)

45.(AB) \( f(x) = a \cos |x^3 - x| + b |x| \sin |x^3 + x| = \begin{cases} a \cos(x - x^3) + bx \sin(x^3 + 2) & 0 \leq x < h \\ a \cos(x^3 - x) + bx \sin(x^3 + x) - h < x < 0 \end{cases} \)

\( f'(x) = \begin{cases} -a \sin(x - x^3)(1 - 3x^2) + b \sin(x^3 + x) + bx \cos(x^3 + x)(3x^2 + 1) & 0 \leq x < h \\ -a \sin(x^3 - x)(3x^2 - 1) + b \sin(x^3 + x) + bx \cos(x^3 + x)(3x^2 + 1) & -h < x < 0 \end{cases} \)

Differentiable at \( x = 0 \)

\( f(x) = \begin{cases} a \cos(x^3 - x) + bx \sin(x^3 + x) & 1 \leq x < 1 + h \\ a \cos(x - x^3) + bx \sin(x^3 + x) & 1 + h < x < 1 \end{cases} \)

Differentiable at \( x = 1 \)

46. Let \( f(x) = \lim_{n \to \infty} \frac{n^n(x + n)(x + n + \frac{n}{2})... (x + n\frac{n}{n})}{n!(x^2 + n^2)(x^2 + n^2 + \frac{n^2}{4})... (x^2 + n^2 + \frac{n^2}{n^2})} \) for all \( x > 0 \). Then
(A) \[ f\left(\frac{1}{2}\right) > f\left(1\right) \]  
(B) \[ f\left(\frac{1}{3}\right) \leq f\left(\frac{2}{3}\right) \]  
(C) \[ f'(2) \leq 0 \]  
(D) \[ \frac{f'(3)}{f(3)} \geq \frac{f'(2)}{f(2)} \]

46.(BC) \[ f(x) = \lim_{n \to \infty} \frac{n^n(x^n+\frac{n}{2}x+\frac{n^2}{4})}{n!(x^2+n^2)^{\frac{n}{2}}}, \quad \forall x > 0 \]

\[ \ln f(x) = x \lim_{n \to \infty} \frac{1}{n} \left[ \ln \left( \frac{n}{1} \right) + \ln \left( \frac{n}{2} \right) + \cdots + \ln \left( \frac{n}{n} \right) \right] = x \left[ \int_0^1 \frac{dt}{t} + \int_0^1 \frac{\sqrt{1+t}}{t} \right] \]

\[ g(x) = \ln f(x) = x \int_0^1 \frac{\ln \left( \frac{ty+1}{y^2+1} \right)}{dt} \]

\[ f(x) = e^{\int_0^x \frac{1}{t} dt} \]

\[ f'(x) = f(x) \times \ln \left( \frac{1+x}{1+x^2} \right) \]

\[ f'(x) > 0 \quad \text{if} \quad x > \frac{1}{\sqrt{1+x^2}} \]

\[ 1 + x^2 < 1 + x \quad \text{for} \quad x > 0, x < 1 \]

(B) correct

\[ \lim_{x \to 2} \frac{f'(x)}{g'(x)} = \lim_{x \to 2} \frac{f'(x)}{g'(x)} = \frac{3}{5} \]

\[ f'(2) = f'(2), \quad \ln \left( \frac{3}{5} \right) < 0 \quad \text{(C) correct.} \]

47. Let \( f : \mathbb{R} \to (0, \infty) \) and \( g : \mathbb{R} \to \mathbb{R} \) be twice differentiable functions such that \( f'' \) and \( g'' \) are continuous functions on \( \mathbb{R} \). Suppose \( f'(2) = g(2) = 0, \quad f''(2) \neq 0 \) and \( g'(2) \neq 0 \). If \( \lim_{x \to 2} \frac{f(x)g'(x)}{f'(x)} = 1 \), then

(A) \( f \) has a local minimum at \( x = 2 \)  
(B) \( f \) has a local maximum at \( x = 2 \)  
(C) \( f''(2) > f'(2) \)  
(D) \( f(x) - f''(x) = 0 \) for at least one \( x \in \mathbb{R} \)
47. (AD) \( \lim_{x \to 2} \frac{f(x)}{g(x)} = 1 \)

\[ \Rightarrow \frac{\lim_{x \to 2} f'(x) g(x) + f(x) g'(x)}{f'(x) g(x) + f''(x) g'(x) + f(x) g''(x)} = 1 \]

\[ \Rightarrow \frac{0 + f(2) g(2)}{0 + f''(2) g(2)} = 1 \]

\[ \Rightarrow f(2) = f''(2) \]

\[ \Rightarrow \text{Option C is incorrect and option D is correct.} \]

Since \( f'(2) = f(2) > 0 \)

\[ \Rightarrow f \text{ has local minima at } x = 2 \]

\[ \Rightarrow \text{Option A is correct.} \]

48. Let \( a, b \in \mathbb{R} \) and \( a^2 + b^2 \neq 0 \). Suppose \( S = \left\{ z \in \mathbb{C} : z = \frac{1}{a + ibt}, t \in \mathbb{R}, t \neq 0 \right\} \), where \( i = \sqrt{-1} \). If \( z = x + iy \) and \( z \in S \), then \((x, y)\) lies on

- (A) the circle with radius \( \frac{1}{2a} \) and centre \( \left( \frac{1}{2a}, 0 \right) \) for \( a > 0, b \neq 0 \)
- (B) the circle with radius \( -\frac{1}{2a} \) and centre \( \left( -\frac{1}{2a}, 0 \right) \) for \( a < 0, b \neq 0 \)
- (C) the x-axis for \( a \neq 0, b = 0 \)
- (D) the y-axis for \( a = 0, b \neq 0 \)

\[ \begin{align*}
\{ z \in c, z = \frac{1}{a + ibt}, t \in \mathbb{R}, t \neq 0 \} = \{ z \in c, z = \frac{a - ibt}{a^2 + b^2 t^2}, t \in \mathbb{R}, t \neq 0 \} \\
\end{align*} \]

As \( z = x + iy \) and \( z \in S \)

\[ \therefore x = \frac{a}{a^2 + b^2 t^2} \quad y = \frac{-bt}{a^2 + b^2 t^2} \]

\[ \Rightarrow bx + ay = 0 \quad \Rightarrow \quad t = \frac{-ay}{bx}, b \neq 0 \]

\[ \therefore x = \frac{a x^2}{a^2 + b^2 \frac{y^2}{b^2} x^2} \quad \Rightarrow \quad x = \frac{a x^2}{x^2 a^2 + a^2 y^2} \]

\[ \Rightarrow a x (x^2 + y^2) = x^2 \quad \Rightarrow \quad x^2 + y^2 - \frac{x}{a} = 0 \]

\[ \Rightarrow \text{circle centre } = \left( \frac{1}{2a}, 0 \right), \text{ radius } = \frac{1}{2a} \text{ for } a > 0; b \neq 0 \]

For \( b = 0, a \neq 0 \) equation (i) becomes, \( y = 0 \quad \Rightarrow \quad \text{lies on x-axis} \)

For \( a = 0, b \neq 0 \) equation (i) becomes, \( x = 0 \quad \Rightarrow \quad \text{lies on y-axis} \)

49. Let \( a, \lambda, \mu \in \mathbb{R} \). Consider the system of linear equations

\[ ax + 2y = \lambda \]

\[ 3x - 2y = \mu \]

Which of the following statement(s) is(are) correct?

- (A) If \( a = -3 \), then the system has infinitely many solutions for all values of \( \lambda \) and \( \mu \)
- (B) If \( a \neq -3 \), then the system has a unique solution for all values of \( \lambda \) and \( \mu \)
- (C) If \( \lambda + \mu = 0 \), then the system has infinitely many solutions for \( a = -3 \)
- (D) If \( \lambda + \mu \neq 0 \), then the system has no solution for \( a = -3 \)

\[ 49. (BCD) \quad \begin{vmatrix} a & 2 \\ 3 & -2 \end{vmatrix} = -2a - 6 = -2(a + 3) \]
\[|A_1| = \begin{vmatrix} \lambda & 2 \\ \mu & -2 \end{vmatrix} = -2(\lambda + \mu)\]
\[|A_2| = \begin{vmatrix} a & \lambda \\ \frac{3}{\mu} & -3 \end{vmatrix} = a\mu - 3\mu\]

For \(a = -3\) \(|A| = 0\) system as infinite many solutions and if \(|A_1| = 0\) and \(|A_2| = 0\)
\[\Rightarrow \lambda + \mu = 0 \Rightarrow \text{Option A is incorrect}\]

If \(\lambda + \mu \neq 0\) and \(a = -3\)
\[\Rightarrow |A| = 0, |A_2| = 0 \Rightarrow \text{System has infinite solutions}\]

For \(\lambda + \mu \neq 0\) and \(a = -3\) \(|A| = 0, |A_2| \neq 0, |A_1| \neq 0\)
\[\Rightarrow \text{System has no solution} \Rightarrow \text{Option D is correct}\]

50. Let \(f : \left[\frac{-1}{2}, 2\right] \rightarrow \mathbb{R}\) and \(g : \left[\frac{-1}{2}, 2\right] \rightarrow \mathbb{R}\) be functions defined by \(f(x) = [x^2 - 3]\) and \(g(x) = |x| f(x) + |4x - 7| f(x)\), where \([y]\) denotes the greatest integer less than or equal to \(y\) for \(y \in \mathbb{R}\). Then:

(A) \(f\) is discontinuous exactly at three points in \(\left[-\frac{1}{2}, 2\right]\)

(B) \(f\) is discontinuous exactly at four points in \(\left[-\frac{1}{2}, 2\right]\)

(C) \(g\) is NOT differentiable exactly at four points in \(\left(-\frac{1}{2}, 2\right)\)

(D) \(g\) is NOT differentiable exactly at five points in \(\left(-\frac{1}{2}, 2\right)\)

50.(BC) \(f(x) = [x^2 - 3]\)
\(g(x) = |x| f(x) + |4x - 7| f(x) = (|x| + |4x - 7|)([x^2] - 3)\)
\(f(x)\) discontinuous at \(\{1, \sqrt{2}, \sqrt{3}, 2\}\)
\[
\begin{vmatrix}
7 - 5x & (-3) \\
7 - 3x & (-3) \\
7 - 3x & (-2) \\
7 - 3x & (-1) \\
5x - 7 & 0
\end{vmatrix}
\begin{vmatrix}
-1 \\
0 \\
1 \\
\sqrt{2} \\
\sqrt{3}
\end{vmatrix}
\begin{vmatrix}
0 \\
1 \\
\frac{7}{4} \\
2
\end{vmatrix}
\begin{vmatrix}
3 \\
x = 2
\end{vmatrix}
\]
Discontinuous at \(\{1, \sqrt{2}, \sqrt{3}, 2\}\)
Non differentiable \(\{0, 1, \sqrt{2}, \sqrt{3}, 2\}\)

**SECTION III (Maximum Marks : 15)**

- This section contains **TWO** paragraph.
- Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:
Paragraph for Questions 51 - 52

Football teams $T_1$ and $T_2$ have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of $T_1$ winning, drawing and losing a game against $T_2$ are $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$, respectively.

Each team gets 3 points for a win, 1 point for a draw and 0 point for a loss in a game. Let $X$ and $Y$ denote the total points scored by teams $T_1$ and $T_2$, respectively, after two games.

51. $P(X > Y)$ is:
   (A) $\frac{1}{4}$  (B) $\frac{5}{12}$  (C) $\frac{1}{2}$  (D) $\frac{7}{12}$

51. (B) For (X > Y)
   Match -1  Match – 2  $P(X > Y)$
   $T_1$ wins  $T_1$ wins  $\frac{1}{2} \times \frac{1}{2}$
   $T_1$ wins  $T_1$ draws  $\frac{1}{2} \times \frac{1}{6}$
   $T_1$ draw  $T_1$ wins  $\frac{1}{6} \times \frac{1}{2}$

$P(X > Y) = \frac{1}{4} + \frac{1}{12} + \frac{1}{12} = \frac{5}{12}$

52. $P(X = Y)$ is:
   (A) $\frac{11}{36}$  (B) $\frac{1}{3}$  (C) $\frac{13}{36}$  (D) $\frac{1}{2}$

52. (C) For (X = Y)
   Match -1  Match – 2  $P(X > Y)$
   $T_1$ wins  $T_1$ Lose  $\frac{1}{2} \times \frac{1}{3}$
   $T_1$ Lose  $T_1$ win  $\frac{1}{3} \times \frac{1}{2}$
   $T_1$ draw  $T_1$ draw  $\frac{1}{6} \times \frac{1}{6}$

$P(X = Y) = \frac{1}{6} + \frac{1}{6} + \frac{1}{36} = \frac{13}{36}$

Paragraph for Questions 53 - 54

Let $F_1(x_1, 0)$ and $F_2(x_2, 0)$, for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{8} = 1$. Suppose a parabola having vertex at the origin and focus at $F_2$ intersects the ellipse at point $M$ in the first quadrant and at point $N$ in the fourth quadrant.

53. The orthocentre of the triangle $F_1MN$ is:
   (A) $\left(\frac{9}{10}, 0\right)$  (B) $\left(\frac{2}{3}, 0\right)$  (C) $\left(\frac{9}{10}, 0\right)$  (D) $\left(\frac{2}{3}, \sqrt{6}\right)$

53. (A) Solving $y^2 = 4x$
   $\frac{x^2}{3} + \frac{4y^2}{8} = 1$
\[ M \left( \frac{3}{2}, \sqrt{6} \right) N \left( \frac{3}{2}, -\sqrt{6} \right) \]

Equation of altitude through \( M \)
\[ y - \sqrt{6} = \frac{5}{2\sqrt{6}} \left( x - \frac{3}{2} \right) \]

According to figure, orthocentre lies on \( x = \) axis
\[ x = -\frac{9}{10} \quad \therefore \quad \text{orthocentre } \left( -\frac{9}{10}, 0 \right) \]

54. If the tangents to the ellipse at \( M \) and \( N \) meet at \( R \) and the normal to the parabola at \( M \) meets the \( x \)-axis at \( Q \), then the ratio of the area of the triangle \( MQR \) to area of the quadrilateral \( MF_1NF_2 \) is:
- (A) 3 : 4
- (B) 4 : 5
- (C) 5 : 8
- (D) 2 : 3

54. (C) Equation of normal at point \( M \)
\[ y = mx - 2m - m^3 ; \quad \text{where } m = -t = -\frac{3}{\sqrt{6}} \]
\[ Q \left( \frac{7}{2}, 0 \right) \]

Equation of tangent at point \( M \) is given by
\[ \frac{x}{9} \cdot \frac{3}{2} + \frac{y}{8} \cdot \sqrt{6} = 1 \]
\[ \frac{x}{9} \cdot \frac{3}{2} - \frac{y}{8} \cdot \sqrt{6} = 1 \]
\[ x = 6, y = 0 \]
\[ M \left( \frac{3}{2}, \sqrt{6} \right) \quad R(6, 0) \]
\[ Q \left( \frac{7}{2}, 0 \right) \quad \text{Area of } \Delta MQR = \frac{5\sqrt{6}}{4} \]

Area of quadrilateral \( MF_1NF_2 \)
\[ \frac{\text{ar. of } \Delta MQR}{\text{ar. of } \text{quad. }} \frac{MF_1NF_2}{5} = \frac{5}{8} \]