JEE Main – 2020

7th January 2020 (Evening Shift)

General Instructions

1. The test is of **3 hours** duration and the maximum marks is **300.**
2. The question paper consists of **3 Parts** (Part I: **Physics**, Part II: **Chemistry**, Part III: **Mathematics**). Each Part has **two** sections (Section 1 & Section 2).
3. **Section 1** contains **20 Multiple Choice Questions**. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE CHOICE** is correct.
4. **Section 2** contains **5 Numerical Value Type Questions**. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

Marking Scheme

1. **Section – 1:** +4 for correct answer, –1 (negative marking) for incorrect answer, 0 for all other cases.
2. **Section – 2:** +4 for correct answer, 0 for all other cases. There is no negative marking.

SUBJECT I: PHYSICS	MARKS: 100

SECTION 1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. Two ideal Carnot engines operate in cascade (all heat given up by one engine is used by the other engine to produce work) between temperatures, T_1 and T_2. The temperature of the hot reservoir of the first engine is T_1 and the temperature of the cold reservoir of the second engine is T_2. T is temperature of the sink of first engine which is also the source for the second engine. How is T related to T_1 and T_2, if both the engines perform equal amount of work?

(1) $T = \dfrac{2T_1T_2}{T_1+T_2}$ **(2)** $T = \sqrt{T_1T_2}$ **(3)** $T = \dfrac{T_1+T_2}{2}$ **(4)** $T = 0$

2. A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much less than the speed of sound). The observer hears 2 beats/sec. The oscillation frequency of each tuning fork is $v_0 = 1400$ Hz and the velocity of sound in air is 350 m/s. The speed of each tuning fork is close to:

(1) 1 m/s **(2)** $\frac{1}{2}$ m/s **(3)** $\frac{1}{4}$ m/s **(4)** $\frac{1}{8}$ m/s

3. A planar loop of wire rotates in a uniform magnetic field. Initially, at $t = 0$, the plane of the loop is perpendicular to the magnetic field. If it rotates with a period of 10 s about an axis in its plane then the magnitude of induced emf will be maximum and minimum, respectively at:

(1) 2.5 s and 7.5 s **(2)** 5.0 s and 10.0 s **(3)** 5.0 s and 7.5 s **(4)** 2.5 s and 5.0 s

4. An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed of 3 m/s. The frictional force opposing the motion is 6000 N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator $(g = 10\ \text{m/s}^2)$ must be at least :

(1) 66000 W **(2)** 62360 W **(3)** 56300 W **(4)** 48000 W

5. Mass per unit area of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre is:

(1) $2\pi a^4\left(\dfrac{A}{4}+\dfrac{B}{5}\right)$ **(2)** $2\pi a^4\left(\dfrac{A}{4}+\dfrac{aB}{5}\right)$

(3) $\pi a^4\left(\dfrac{A}{4}+\dfrac{aB}{5}\right)$ **(4)** $2\pi a^4\left(\dfrac{aA}{4}+\dfrac{B}{5}\right)$

6. The dimension of $\dfrac{B^2}{2\mu_0}$, where B is magnetic field and μ_0 is the magnetic permeability of vacuum, is:

(1) ML^2T^{-2} **(2)** $ML^{-1}T^{-2}$ **(3)** ML^2T^{-1} **(4)** MLT^{-2}

7. The activity of a radioactive sample falls from $700\ s^{-1}$ to $500\ s^{-1}$ in 30 minutes. Its half life is close to:

(1) 66 min **(2)** 62 min **(3)** 72 min **(4)** 52 min

8. In a building there are 15 bulbs of 45 W, 15 bulbs of 100 W, 15 small fans of 10 W and 2 heaters of 1 kw. The voltage of electric main in 220 V. The minimum fuse capacity (rated value) of the building will be:

(1) 25 A **(2)** 15 A **(3)** 10 A **(4)** 20 A

9. A thin lens made of glass (refractive index = 1.5) of focal length f = 16 cm is immersed in a liquid of refractive index 1.42. If its focal length in liquid is f_1, then the ratio f_1 / f is closest to the integer:

(1) 1 **(2)** 9 **(3)** 17 **(4)** 5

10. In a Young's double slit experiment, the separation between the slits is 0.15 mm. In the experiment, a source of light of wavelength 589 nm is used and the interference pattern is observed on a screen kept 1.5 m away. The separation between the successive bright fringes on the screen is:

(1) 3.9 mm **(2)** 4.9 mm **(3)** 5.9 mm **(4)** 6.9 mm

11. An electron (of mass m) and a photon have the same energy E in the range of a few eV. The ratio of the de-Broglie wavelength associated with the electron and the wavelength of the photon is (c = speed of light in vacuum)

(1) $\frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$ **(2)** $\left(\frac{E}{2m}\right)^{1/2}$ **(3)** $c(2mE)^{1/2}$ **(4)** $\frac{1}{c}\left(\frac{2E}{m}\right)^{1/2}$

12. The electric field of a plane electromagnetic wave is given by

$$\vec{E} = E_0 \frac{\hat{i}+\hat{j}}{\sqrt{2}} \cos(kz + \omega t)$$

At $t = 0$, a positively charged particle is at the point $(x, y, z) = \left(0, 0, \frac{\pi}{k}\right)$. If its instantaneous velocity at $(t = 0)$ is $v_0\hat{k}$, the force acting on it due to the wave is:

(1) zero **(2)** antiparallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

(3) parallel to $\hat{k}$ **(4)** parallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

13. An emf of 20 V is applied at time t = 0 to a circuit containing is series 10 mH inductor and $5\,\Omega$ resister. The ratio of the currents at time $t = \infty$ and at $t = 40s$ is close to : (Take $e^2 = 7.389$)

(1) 1.06 **(2)** 1.15 **(3)** 0.84 **(4)** 1.46

14. Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently the mean collision time between the gas molecule changes from τ_1 to τ_2. If $\frac{C_p}{C_v} = \gamma$ for this gas then a good estimate for $\frac{\tau_2}{\tau_1}$ is given by:

(1) $\frac{1}{2}$ **(2)** 2 **(3)** $\left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$ **(4)** $\left(\frac{1}{2}\right)^{\gamma}$

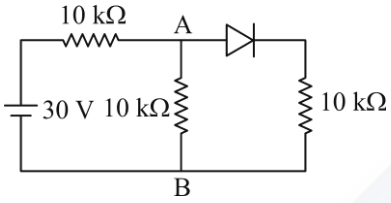
15. An ideal fluid flows (laminar flow) through a pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and the maximum velocities of flued in this pipe is:

(1) $\frac{3}{4}$ **(2)** $\frac{9}{16}$ **(3)** $\frac{81}{256}$ **(4)** $\frac{\sqrt{3}}{2}$

16. A particle of mass m and charge q has an initial velocity $\vec{v} = v_0\hat{j}$. If an electric field $\vec{E} = E_0\hat{i}$ and magnetic field $\vec{B} = B_0\hat{i}$ act on the particle, its speed will double after a time:

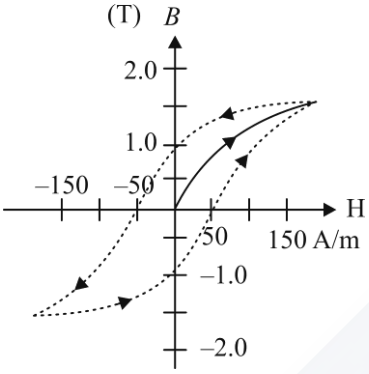
(1) $\frac{\sqrt{3}mv_0}{qE_0}$ **(2)** $\frac{3mv_0}{qE_0}$ **(3)** $\frac{\sqrt{2}mv_0}{qE_0}$ **(4)** $\frac{2mv_0}{qE_0}$

17. In the figure, potential difference between A and B is:



(1) 5 V **(2)** 15 V **(3)** 10 V **(4)** zero

18.



The figure gives experimentally measured B vs. H variation in a ferromagnetic material. The retentivity, co-ercivity and saturation, respectively, of the material are:

(1) 1.5 T, 50 A/m and 1.0 T **(2)** 150 A/m, 1.0 T and 1.5 T
(3) 1.0 T, 50 A/m and 1.5 T **(4)** 1.5 T, 50 A/m and 1.0 T

19. A mass of 10 kg is suspended by a rope of length 4 m, from the ceiling. A force F is applied horizontally at the mid-point of the rope such that the top half of the rope makes an angle of 45° with te vertical. Then F equals: (Take $g = 10\ \text{ms}^{-2}$ and the rope to be massless)

(1) 90 N **(2)** 100 N **(3)** 70 N **(4)** 75 N

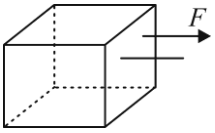
20. A box weights 196 N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to (Take $g = 10\ \text{ms}^{-2}$ at the north pole and the radius of the earth $= 6400$ km) :

(1) 194.66 N **(2)** 195.66 N **(3)** 195.32 N **(4)** 194.32 N

SECTION 2

This section has FIVE (05) Questions. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

21. The balancing length for a cell is 560 cm in a potentiometer experiment. When an external resistance of $10\ \Omega$ is connected in parallel to the cell, the balancing length changes by 60 cm. If the internal resistance of the cell is $\frac{N}{10}\Omega$, where N is an integer the value of N is:

22. 

Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4,$ the maximum possible value of $100 \times \frac{b}{a}$ for box not to topple before moving is _____ .

23. The sum of two forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ such that $|\vec{R}| = |\vec{P}|$. The angle θ (in degrees) that the resultant of $2\vec{P}$ and $\vec{Q}$ will make with $\vec{Q}$ is, _____ .

24. M grams of steam at 100°C is mixed with 200 g of ice at its melting point in a thermally insulated container. If it produces liquid water at 40°C [heat of vaporization of water is 540 cal/g and heat of fusion of ice is 80 cal/g], the value of M is _____ .

25. A 60pF capacitor is fully charged by a 20 V supply. It is then disconnected from the supply and is connected to another uncharged 60 pF capacitor in parallel. The electrostatic energy that is lost in this process by the time the charge is redistributed between them is (in nJ) _____ .

SUBJECT II: CHEMISTRY	MARKS: 100

SECTION 1

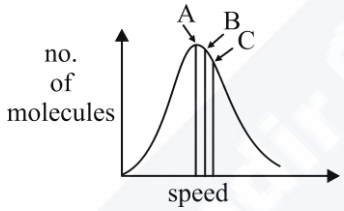
This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. The bond order and the magnetic characteristics of CN^- are:

(1) 3, diamagnetic **(2)** $2\frac{1}{2}$, diamagnetic

(3) $2\frac{1}{2}$, paramagnetic **(4)** 3, paramagnetic

2. Identify the correct labels of A, B and C in the following graph from the options given below:



Root mean square speed (V_{rms}); most probable speed (V_{mp}); Average speed (V_{av})

(1) A-V_{av}; B-V_{rms}; C-V_{mp} **(2)** A-V_{mp}; B-V_{av}; C-V_{rms}

(3) A-V_{mp}; B-V_{rms}; C-V_{av} **(4)** A-V_{rms}; B-V_{mp}; C-V_{av}

3. Which of the following statements is correct?

(1) Gluconic acid is a partial oxidation product of glucose

(2) Gluconic acid is obtained by oxidation of glucose with HNO_3

(3) Gluconic acid can form cyclic (acetal/ hemiacetal) structure

(4) Gluconic acid is a dicarboxylic acid

4. Among the statements (a)-(d), the incorrect ones are:

(a) Octahedral Co(III) complexes with strong field ligands have very high magnetic moments

(b) When $\Delta_0 < P$, the d-electron configuration of Co(III) in an octahedral complex is $t_{2g}^4\ e_g^2$

(c) Wavelength of light absorbed by $[Co(en)_3]^{3+}$ is lower than that of $[CoF_6]^{3-}$

(d) If the Δ_0 for an octahedral complex of Co(III) is 18,000 cm^{-1}, the Δ_t for its tetrahedral complex with the same ligand will be 16,000 cm^{-1}

(1) (a) and (b) only **(2)** (a) and (d) only **(3)** (b) and (c) only **(4)** (c) and (d) only

5. Two open beakers one containing a solvent and the other containing a mixture of that solvent with a non volatile solute are together sealed in a container. Over time:

(1) the volume of the solution and the solvent does not change

(2) the volume of the solution increases and the volume of the solvent decreases

(3) the volume of the solution decreases and the volume of the solvent increases

(4) the volume of the solution does not change and the volume of the solvent decreases

6. The ammonia (NH_3) released on quantitative reaction of 0.6 g urea (NH_2CONH_2) with sodium hydroxide (NaOH) can be neutralized by:

(1) 200 ml of 0.4 N HCl **(2)** 100 ml of 0.1 N HCl

(3) 100 ml of 0.2 N HCl **(4)** 200 ml of 0.2 N HCl

7. The equation that is incorrect is:

(1) $\left(\Lambda_m^0\right)_{NaBr}-\left(\Lambda_m^0\right)_{NaCl}=\left(\Lambda_m^0\right)_{KBr}-\left(\Lambda_m^0\right)_{KCl}$

(2) $\left(\Lambda_m^0\right)_{H_2O}=\left(\Lambda_m^0\right)_{HCl}+\left(\Lambda_m^0\right)_{NaOH}-\left(\Lambda_m^0\right)_{NaCl}$

(3) $\left(\Lambda_m^0\right)_{NaBr}-\left(\Lambda_m^0\right)_{NaI}=\left(\Lambda_m^0\right)_{KBr}-\left(\Lambda_m^0\right)_{NaBr}$

(4) $\left(\Lambda_m^0\right)_{KCl}-\left(\Lambda_m^0\right)_{NaCl}=\left(\Lambda_m^0\right)_{KBr}-\left(\Lambda_m^0\right)_{NaBr}$

8. Among statements (a)-(d), the correct ones are:

(a) Decomposition of hydrogen peroxide gives dioxygen.

(b) Like hydrogen peroxide, compounds, such as $KClO_3, Pb(NO_3)_2$ and $NaNO_3$ when heated liberate dioxygen.

(c) 2-Ethylanthraquinone is useful for the industrial preparation of hydrogen peroxide.

(d) Hydrogen peroxide is used for the manufacture of sodium perborate.

(1) (a) and (c) only **(2)** (a), (b) and (c) only

(3) (a), (c) and (d) only **(4)** (a), (b), (c) and (d)

9. For the reaction $2H_2(g)+2NO(g)\rightarrow N_2(g)+2H_2O(g)$ the observed rate expression is, rate $=k_f[NO]^2[H_2]$. The rate expression for the reverse reaction is:

(1) $k_b[N_2][H_2O]^2/[H_2]$ **(2)** $k_b[N_2][H_2O]$

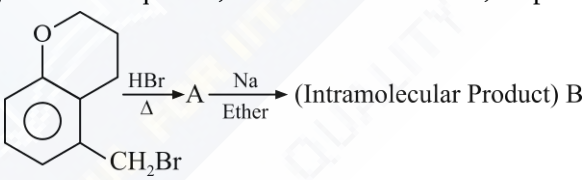
(3) $k_b[N_2][H_2O]^2/[NO]$ **(4)** $k_b[N_2][H_2O]^2$

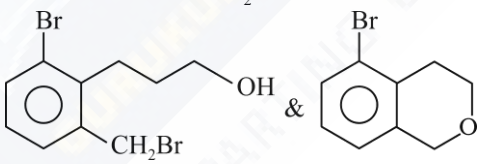
10. The number of possible optical isomers for the complexes MA_2B_2 with sp^3 and dsp^2 hybridized metal atom, respectively, is:

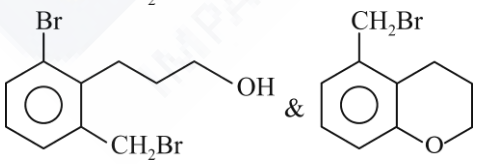
Note: A and B are unidentate neutral and unidentate monoanionic ligands, respectively.

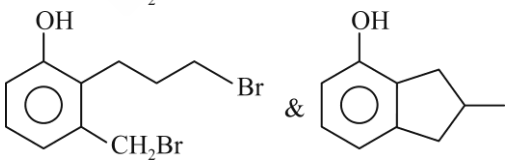
(1) 0 and 0 **(2)** 0 and 1 **(3)** 2 and 2 **(4)** 0 and 2

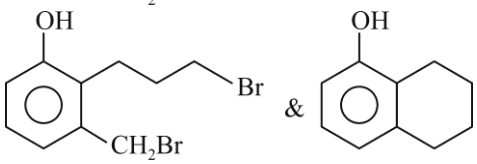
11. In the following reaction sequence, structures of A and B, respectively will be:



(1) 

(2) 

(3) 

(4) 

12. A chromatography column, packed with silica gel as stationary phase, was used to separate a mixture of compounds consisting of (A) benzanilide (B) aniline and (C) acetophenone. When the column is eluted with a mixture of solvents, hexane: ethyl acetate (20 : 80), the sequence of obtained compounds is:

(1) (A), (B) and (C) **(2)** (B), (C) and (A)

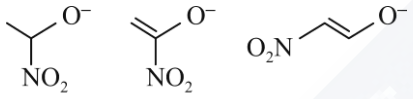
(3) (C), (A) and (B) **(4)** (B), (A) and (C)

13. The refining method used when the metal and the impurities have low and high melting temperatures, respectively, is:

(1) distillation **(2)** liquation

(3) zone refining **(4)** vapour phase refining

14. The correct order of stability for the following alkoxides is:



(A) (B) (C)

(1) (B) > (A) > (C) **(2)** (B) > (C) > (A) **(3)** (C) > (B) > (A) **(4)** (C) > (A) > (B)

15. The redox reaction among the following is:

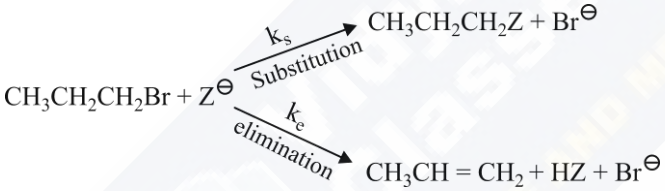
(1) reaction of H_2SO_4 with NaOH

(2) formatin of ozone from atmospheric oxygen in the presence of sunlight

(3) combination of dinitrogen with dioxygen at 2000 K

(4) reaction of $[Co(H_2O)_6]Cl_3$ with $AgNO_3$

16. For the following reactions



where,

$Z^{\ominus} = CH_3CH_2O^{\ominus}$ (A) or $H_3C-\underset{\displaystyle CH_3}{\overset{\displaystyle CH_3}{\underset{|}{\overset{|}{C}}}}-O^{\ominus}$ (B),

k_s and $k_{e'}$ are, respectively, the rate constants for substitution and elimination, and $\mu = \frac{k_s}{k_e}$, the correct option is:

(1) $\mu_B > \mu_A$ and $k_e(B) > K_e(A)$ **(2)** $\mu_A > \mu_B$ and $k_e(B) > K_e(A)$

(3) $\mu_A > \mu_B$ and $k_e(A) > K_e(B)$ **(4)** $\mu_B > \mu_A$ and $k_e(A) > K_e(B)$

17. Within each pair of elements F & Cl, S & Se, and Li & Na, respectively, the elements that release more energy upon an electron gain are:

(1) F, Se and Na **(2)** F, S and Li **(3)** Cl, S and Li **(4)** Cl, Se and Na

18. In the following reactions, products (A) and (B), respectively, are:

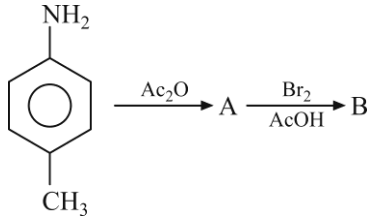
$$\underset{\text{(hot and conc.)}}{NaOH} + Cl_2 \rightarrow (A) + \text{side products}$$

$$\underset{\text{(dry)}}{Ca(OH)_2} + Cl_2 \rightarrow (B) + \text{side products}$$

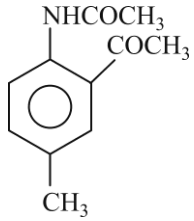
(1) $NaClO_3$ and $Ca(ClO_3)_2$ **(2)** $NaClO_3$ and $Ca(OCl)_2$

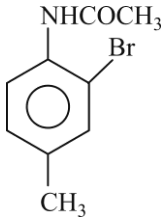
(3) NaOCl and $Ca(ClO_3)_2$ **(3)** NaOCl and $Ca(OCl)_2$

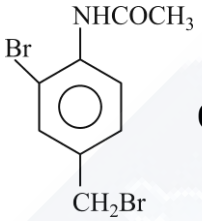
19. In the following reaction sequence,

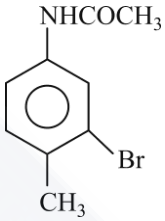


the major product B is:

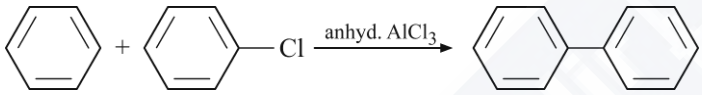
(1) 

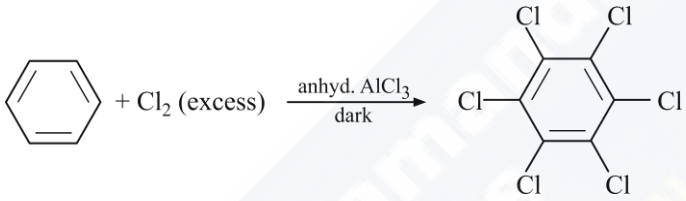
(2) 

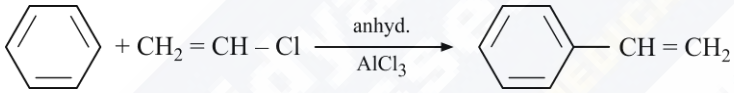
(3) 

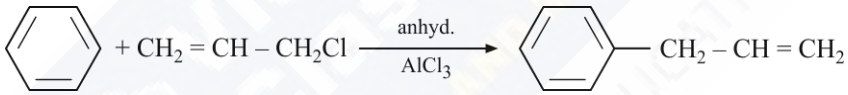
(4) 

20. Consider the following reactions :

(a) 

(b) 

(c) 

(d) 

Which of these reactions are possible?

(1) (a) and (d) **(2)** (b), (c) and (d) **(3)** (b) and (d) **(4)** (a) and (b)

SECTION 2

This section has FIVE (05) Questions. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

21. The standard heat of formation $(\Delta_f H^0_{298})$ of ethane (in kJ/mol), if the heat of combustion of ethane, hydrogen and graphite are –1560, –393.5 and –286 kJ/mol, respectively is _____ .

22. Consider the following reactions:

$$NaCl + K_2Cr_2O_7 + \underset{\text{(Conc.)}}{H_2SO_4} \rightarrow (A) + \text{side products}$$

$$(A) + NaOH \rightarrow (B) + \text{side products}$$

$$(B) + \underset{\text{(dilute)}}{H_2SO_4} + H_2O_2 \rightarrow (C) + \text{Side products}$$

The sum of the total number of atoms in one molecule each of (A), (B) and (C) is _____.

23. The flocculation value of HCl for arsenic sulphide sol. is 30 m mol L^{-1}. If H_2SO_4 is used for the flocculation of arsenic sulphide, the amount, in grams, of H_2SO_4 in 250 ml required for the above purpose is _____ . (molecular mass of $H_2SO_4 = 98$ g / mol)

24. The number of sp^2 hybridized carbons present in "Aspartame" is _____.

25. 3 g of acetic acid is added to 250 mL of 0.1 M HCl and the solution made up to 500 mL . To 20 mL of this solution $\frac{1}{2}$ mL of 5 M NaOH is added. The pH of the solution is _____.

[Given: pKa of acetic acid $= 4.75$, molar mass of acetic acid $= 60$ g/mol, $\log 3 = 0.4771$]

Neglect any changes in volume.

SUBJECT III: MATHEMATICS	MARKS: 100

SECTION 1

This section contains 20 Multiple Choice Questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE CHOICE is correct.

1. If $\frac{3+i\sin\theta}{4-i\cos\theta}$, $\theta\in[0, 2\pi]$, is a real number, then an argument of $\sin\theta+i\cos\theta$ is:

(1) $\pi-\tan^{-1}\left(\frac{3}{4}\right)$ **(2)** $\tan^{-1}\left(\frac{4}{3}\right)$ **(3)** $-\tan^{-1}\left(\frac{3}{4}\right)$ **(4)** $\pi-\tan^{-1}\left(\frac{4}{3}\right)$

2. Let the tangents drawn from the origin to the circle, $x^2+y^2-8x-4y+16=0$ touch it at the points A and B. The $(AB)^2$ is equal to:

(1) $\frac{52}{5}$ **(2)** $\frac{64}{5}$ **(3)** $\frac{32}{5}$ **(4)** $\frac{56}{5}$

3. The value of c in the Lagrange's mean value theorem for the function $f(x)=x^3-4x^2+8x+11$, where $x\in[0, 1]$ is:

(1) $\frac{\sqrt{7}-2}{3}$ **(2)** $\frac{4-\sqrt{7}}{3}$ **(3)** $\frac{2}{3}$ **(4)** $\frac{4-\sqrt{5}}{3}$

4. If $3x+4y=12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{9}=1$ for some a $\in R$, then the distance between the foci of the ellipse is:

(1) $2\sqrt{5}$ **(2)** $2\sqrt{7}$ **(3)** 4 **(4)** $2\sqrt{2}$

5. The locus of the mid-points of the perpendiculars drawn from points on the line, $x=2y$ to the line $x=y$ is :

(1) $2x-3y=0$ **(2)** $7x-5y=0$ **(3)** $5x-7y=0$ **(4)** $3x-2y=0$

6. The coefficient of x^7 in the expression $(1+x)^{10}+x(1+x)^9+x^2(1+x)^8+...+x^{10}$ is :

(1) 210 **(2)** 120 **(3)** 420 **(4)** 330

7. Let $A=[a_{ij}]$ and $B=[b_{ij}]$ be two 3 × 3 matrices such that $b_{ij}=(3)^{(i+j-2)}a_{ji}$, where $i, j=1,2,3$. If the determinant of B is 81 then the determinant of A is:

(1) 3 **(2)** 1/3 **(3)** 1/81 **(4)** 1/9

8. Let A, B, C and D be four non-empty sets. The contrapositive statement of "If $A\subseteq B$ and $B\subseteq D$, then $A\subseteq C''$ is:

(1) If $A\not\subseteq C$, then $A\subseteq B$ and $B\subseteq D$ **(2)** If $A\subseteq C$, then $B\subset A$ or $D\subset B$

(3) If $A\not\subseteq C$, then $A\not\subseteq B$ or $B\not\subseteq D$ **(4)** If $A\not\subseteq C$, then $A\not\subseteq B$ and $B\not\subset D$

9. If the sum of the first 40 terms of the series, 3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + …. is (102)m, then m is equal to:

(1) 20 **(2)** 5 **(3)** 10 **(4)** 25

10. The value of α for which $4\alpha\int_{-1}^{2} e^{-\alpha|x|}dx = 5,$ is:

(1) $\log_e\left(\frac{4}{3}\right)$ **(2)** $\log_e\left(\frac{3}{2}\right)$ **(3)** $\log_e 2$ **(4)** $\log_e\sqrt{2}$

11. The area (in square units) of the region $\{(x, y) \in R^2 \mid 4x^2 \le y \le 8x+12\}$ is :

(1) $\frac{128}{3}$ **(2)** $\frac{127}{3}$ **(3)** $\frac{124}{3}$ **(4)** $\frac{125}{3}$

12. Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k, k \ge 1,$ then which one of the following statements is not true?

(1) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$ **(2)** $p_5 = p_2 \cdot p_3$

(3) $p_3 = p_5 - p_4$ **(4)** $p_5 = 11$

13. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k,$ then k is equal to:

(1) $\frac{17}{4}$ **(2)** $\frac{17}{8}$ **(3)** 4 **(4)** $\frac{17}{2}$

14. The number of ordered pairs (r, k) for which $6\cdot{}^{35}C_r = (k^2-3)\cdot{}^{36}C_{r+1},$ where k is an integer, is:

(1) 3 **(2)** 4 **(3)** 2 **(4)** 6

15. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $(0, 2\pi) - \{\pi\}$ which satisfy the equation, $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0,$ then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta \, d\theta$ is equal to:

(1) $\frac{2\pi}{3}$ **(2)** $\frac{\pi}{3} + \frac{1}{6}$ **(3)** $\frac{\pi}{9}$ **(4)** $\frac{\pi}{3}$

16. Let $y = y(x)$ be a function of x satisfying $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and $y\left(\frac{1}{2}\right) = -\frac{1}{4}$. Then $\frac{dy}{dx}$ at $x = \frac{1}{2},$ is equal to:

(1) $\frac{2}{\sqrt{5}}$ **(2)** $-\frac{\sqrt{5}}{2}$ **(3)** $-\frac{\sqrt{5}}{4}$ **(4)** $\frac{\sqrt{5}}{2}$

17. Let $f(x)$ be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim_{x\to 0}\left(2 + \frac{f(x)}{x^3}\right) = 4,$ then which one of the following is not true ?

(1) $f(1) - 4f(-1) = 4$

(2) $x = 1$ is a point of maxima and $x = -1$ is a point of minimum of f.

(3) f is an odd function.

(4) $x = 1$ is a point of minima and $x = -1$ is a point of maxima of f.

18. Let $a_1, a_2, a_3, \ldots$ be a G.P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to:

(1) -171 **(2)** $\frac{511}{3}$ **(3)** -513 **(4)** 171

19. Let $y = y(x)$ be the solution curve of the differential equation, $\left(y^2 - x\right)\frac{dy}{dx} = 1$, satisfying $y(0) = 1$. This curve intersects the x-axis at a point whose abscissa is:

(1) $2 - e$ **(2)** $2 + e$ **(3)** 2 **(4)** $-e$

20. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. If $\lambda = \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ and $\vec{d} = \vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}$, then the ordered pair, $(\lambda, \vec{d})$ is equal to :

(1) $\left(\frac{3}{2}, 3\vec{a}\times\vec{c}\right)$ **(2)** $\left(\frac{3}{2}, 3\vec{b}\times\vec{c}\right)$ **(3)** $\left(-\frac{3}{2}, 3\vec{a}\times\vec{b}\right)$ **(4)** $\left(-\frac{3}{2}, 3\vec{c}\times\vec{b}\right)$

SECTION 2

This section has FIVE (05) Questions. The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

21. If the system of linear equations,

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$3x + 2y + \lambda z = \mu$$

has more than two solutions, then $\mu - \lambda^2$ is equal to _____ .

22. If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then $x \cdot y$ is equal to _____ .

23. If the function f defined on $\left(-\frac{1}{3}, \frac{1}{3}\right)$ by $f(x) = \begin{cases} \frac{1}{x}\log_e\left(\frac{1+3x}{1-2x}\right), & \text{when } x \neq 0 \\ k, & \text{when } x = 0 \end{cases}$ is continuous, then k is equal to _____ .

24. If the foot of the perpendicular drawn from the point (1, 0, 3) on a line passing through $(\alpha, 7, 1)$ is $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$, then α is equal to _____ .

25. Let $X = \{n \in N : 1 \le n \le 50\}$. If $A = \{n \in X : n$ is a multiple of 2$\}$ and $B = \{n \in X : n$ is a multiple of 7$\}$, then the number of elements in the smallest subset of X containing both A and B is _____ .