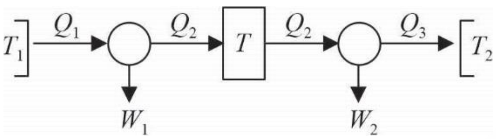
SOLUTIONS

JEE Main – 2020 | 7th January 2020 (Evening Shift)

PHYSICS

SECTION – 1

1.(3)



$W_1 = W_2 \Rightarrow Q_1 - Q_2 = Q_2 - Q_3$

$\Rightarrow \quad T_1 - T = T - T_2 \quad \Rightarrow \quad T = \dfrac{T_1 + T_2}{2}$

2.(3) Beats $= f_0\left(\dfrac{v}{v - v_x}\right) - f_0\left(\dfrac{v}{v + v_s}\right)$

$2 = 1400\left(\dfrac{350}{350 - v_s}\right) - 1400\left(\dfrac{350}{350 + v_s}\right) \quad \Rightarrow \quad v_s = \dfrac{1}{4} m/s$

3.(4) Flux as a function of time

$\phi = BA\cos\ \omega t$

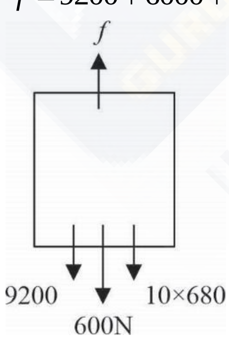
$\varepsilon = \left|\dfrac{d\phi}{dt}\right| = |BA\omega \sin \omega t|$

Magnitude of induced emf will be maximum when

$\sin \omega t = 1 \quad \Rightarrow \quad t = \dfrac{T}{4} = 2.5s.$

It will be minimum when $\sin \omega t = 0 \ \Rightarrow \ t = 5s$

4.(1) $f = 9200 + 6000 + 6800 = 2200N$



Power delivered $= f \times v = 22000 \times 3 = 66000W$

5.(2) $dA = 2\pi r dr$

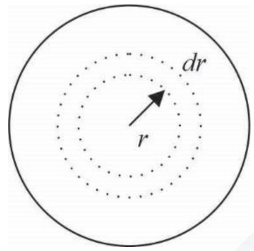
$dm = \sigma dA$

$dI = dm r^2$

$= (A + Br) 2\pi r dr . r^2$

$= 2\pi \int_0^a \left(Ar^3 + Br^4 \right) dr$

$= 2\pi a^4 \left(\frac{A}{4} + \frac{aB}{5} \right)$



6.(2) $\frac{B^2}{2\mu_0} =$ Energy density $= \frac{Energy}{volume} = \frac{\left[ML^2T^{-2}\right]}{\left[L^3\right]} = \left[ML^{-1}T^{-2}\right]$

7.(2) $A = A_0 e^{-\lambda t}$. $500 = 700 e^{-30\lambda}$

$\lambda = \frac{\ln(4/5)}{30} \text{ min}^{-1}$; $t_{1/2} = \frac{0.693}{\lambda} \approx 62 \text{ min}$.

8.(4) Total power consumption $= 15 \times 45 + 15 \times 100 + 15 \times 10 + 2 \times 1000 = 4325W$.

Voltage $= 220V$

Current $= \frac{power}{voltage} = \frac{4325}{220} = 19.66A$ $\Rightarrow$ minimum fuse capacity $= 20A$

9.(2) $\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1 \right) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$

In air

$\frac{1}{f} = \left(\frac{1.5}{1} - 1 \right) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$ ….(1)

In liquid

$\frac{1}{f_L} = \left(\frac{1.5}{1.42} - 1 \right) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$ …..(2)

From (1) & (2)

$\frac{f_2}{f} = \frac{1.5 - 1}{\frac{1.5}{1.42} - 1} \approx 9$

10.(3) $\beta = \frac{\lambda D}{d} = \frac{589 \times 10^{-9} \times 1.5}{15 \times 10^{-5}} = 589 \times 10^{-5} m = 5.9mm$

11.(1) Wavelength of electron, $\lambda_e = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

Wavelength of photon, $\lambda_p = \frac{hc}{E}$

$\frac{\lambda_e}{\lambda_p} = \frac{1}{c} \sqrt{\frac{E}{2m}}$

12.(2) Electric field at $\left(0,0,\frac{\pi}{k}\right)$ is $-E_0\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

Hence force due to electric field is along $-\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

$$\vec{B}=\vec{E}\times\vec{v}=\left[-\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)\times\hat{k}\right]E_0v_0=E_0v_0\left[\frac{-\hat{i}+\hat{j}}{\sqrt{2}}\right]$$

Force due to magnitude filed is along $\vec{v}\times\vec{B}=\hat{k}\times\left(\frac{-\hat{i}+\hat{j}}{\sqrt{2}}\right)=-\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

Hence the net force is anti-parallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

13.(1) $\tau=\frac{L}{R}=\frac{10\times10^{-3}}{5}=2\times10^{-3}\,\text{sec}$

as $40s >>> \tau \Rightarrow I=\frac{E}{R}(1-e^{-t\tau})=\frac{E}{R}$

Note: If $L=10H,\ \tau=2\text{sec}$

$$I=\frac{E}{R}(1-e^{-2})=0.86\frac{E}{R}$$

$$\frac{I_\infty}{I}=\frac{1}{0.86}=1.15$$

14.(3) $\tau=\frac{\text{mean free path}}{V_{\text{rms}}}=\frac{1}{\sqrt{2}n\pi d^2}\frac{1}{V_{\text{rms}}}$

As $n=\frac{N}{V}$ & $V_{\text{rms}}=\sqrt{\frac{3RT}{M}}$

$\therefore \quad \tau\propto\frac{V}{\sqrt{T}}$

Also $TV^{\gamma-1}=\text{constant}\ \Rightarrow T\propto V^{1-\gamma}$

$\therefore \quad \tau\propto\frac{V}{V^{\frac{1-\gamma}{2}}}\propto V^{\frac{\gamma+1}{2}}$

$$\frac{\tau_2}{\tau_1}=\left(\frac{V_2}{V_1}\right)^{\frac{\gamma+1}{2}}=2^{\frac{\gamma+1}{2}}$$

No option matches but reciprocal of the answer is given as correct answer.

15.(2) By equation of continuity:-

$A_1V_1 = A_2V_2$

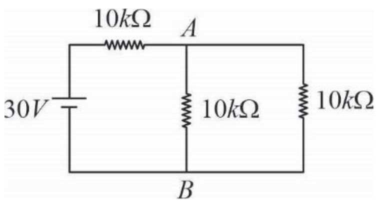
$\frac{V_1}{V_2} = \frac{A_2}{A_1} = \frac{{d_2}^2}{d_1^2}$; $\quad \frac{V_1}{V_2} = \frac{9}{16}$

16.(1) Speed perpendicular to x axis will remain V_0

$(2V_0)^2 = V_0^2 + V_x^2$; $\quad V_x = \sqrt{3}\,V_0$

$V_x = 0 + \frac{qE_0}{m}t$; $\quad t = \frac{\sqrt{3}\,mV_0}{qE_0}$

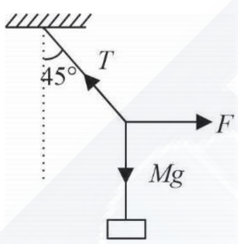
17.(3) Diode is in forward Bias



$V_{AB} = \left(\frac{5}{5+10}\right) \times 30 = 10V$

18.(3)

19.(2)



$T\cos 45° = Mg$

$T\sin 45° = F$

$F = Mg = 100N$

20.(3) $W' = Mg - MR\omega^2$

$\omega = \left(\frac{2\pi}{24 \times 3600}\right) Rad/\sec$

$W' = 195.32N$

SECTION – 2

21.(12) $r = \left(\frac{l - l^1}{l}\right)R$

$= \frac{560 - 500}{500} \times 10$; $\quad r = \frac{12}{10}\Omega$

22.(50) For toppling: $F\left(\frac{a}{2}+b\right) > mg\frac{a}{2} \Rightarrow F > \frac{mga}{a+2b}$

For sliding = $F > umg$

For no toppling before sliding, $umg < \frac{mga}{a+2b}$

$\frac{a+2b}{a} < \frac{1}{\mu} \Rightarrow 1+2\frac{b}{a} < \frac{5}{2} \Rightarrow \frac{b}{a} < 0.75$

But $b_{\max} = 0.5a$ $\quad\quad \therefore \ (100\times\frac{b}{a})_{\max} = 50$

23.(90) $p^2 = p^2 + Q^2 + 2PQ\cos\theta$

$2P\cos\theta + q = 0$

$\tan\alpha = \frac{2P\sin\theta}{(2P\cos\theta+q)} = \infty$

$\alpha = 90$

24.(40) Heat lost by the steam = Heat gained by the ice

$M\times540 + M\times1\times(100-40) = 200\times80 + 200(40-0)$

$M = 40\,gm$

25.(6) $Q_{net} = 1200\,pC$

After connecting the charge will divide equally as capacitance are same

$Q_1 = Q_2 = 600\,pC$

$H = \frac{1}{2}\frac{(1200)^2}{60} - 2\times\frac{1}{2}\frac{(600)^2}{60}$

$= 6000\,pJ$

$H = 6\,nJ$

CHEMISTRY

SECTION – 1

1.(1) CN^- is a 14 electron system.

2.(2) $V_{rms} > V_{avg} > V_{mps}$

3.(1) Gluconic acid is obtained by partial oxidation of glucose by Br_2 / H_2O.

$CH_2(OH)-CH(OH)-CH(OH)-CH(OH)-CH(OH)-COOH$

4.(2) In presence of strong ligand pairing of electron occurs and magnetic moment is generally low.

$\Delta_t = \frac{4}{9}\Delta_0$

5.(2) The Vapour pressure of the beaker that contain pure solvent will be higher. So, the vapours will migrate to the solution beaker.

6.(3) $NH_2CONH_2 + NaOH \rightarrow 2NH_3 + Na_2CO_3$

$\frac{n_{urea}}{1} = \frac{n_{NH_3}}{2}$ ...(1)

$n_{NH_3} = n_{HCl}$ ...(2)

From (1) and (2)

$n_{HCl} = 2 \times n_{urea}$

$= 0.02$ moles.

7.(3) $\left(\wedge_m^0\right)_{NaBr} - \left(\wedge_m^0\right)_{NaI}$ will be equivalent to $\left(\wedge_m^0\right)_{Br^-} - \left(\wedge_m^0\right)_{I^-}$

8.(4) Theory.

In option (C) instead of 2-ethyl anthraquinone, 2-ethyl anthraquinol should have been given.

9.(1) $2H_2(g) + 2NO(g) \longrightarrow N_2(g) + 2H_2O(g)$

rate $= K_f[NO]^2[H_2]$

$k_{eq} = \frac{k_f}{k_b} = \frac{[N_2][H_2O]^2}{[NO]^2[H_2]^2}$

At equilibrium, $r_f = r_b$

$k_f[H_2][NO]^2 = \frac{k_b[N_2][H_2O]^2}{[H_2]}$

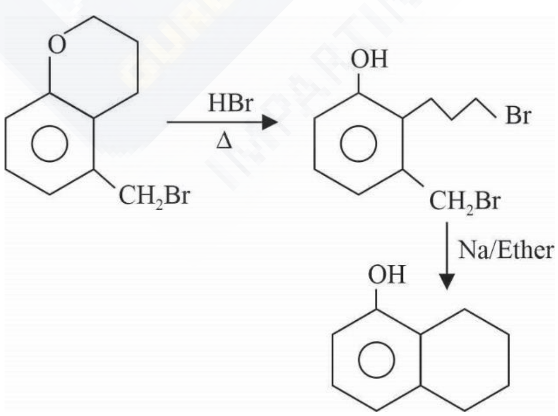
Hence rate expression for reverse rxn

rate $= kb\frac{[N_2][H_2O]^2}{[H_2]}$

10.(1) In sp^3 hybridization in MA_2B_2 there is non chiral centre.

In dsp^2 hybridization both cis and transform has plane of symmetry.

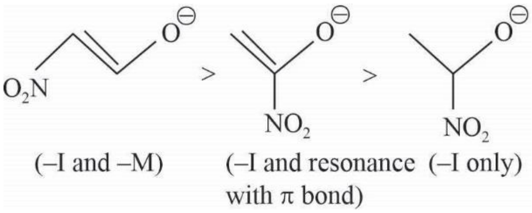
11.(4)



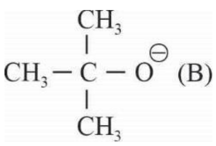
12.(3) Theory based

13.(2) Theory based

14.(3)



15.(3) $N_2 + O_2 \rightarrow 2NO(\text{change in Oxidation state})$

16.(2) With 

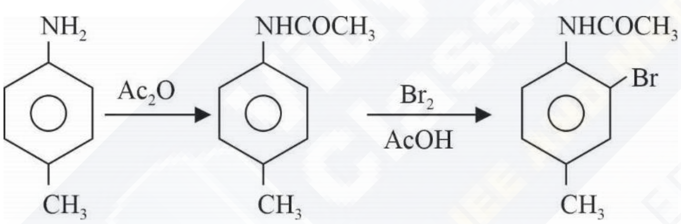
elimination will be fast and with $CH_3 - CH_2 - O^-$ (A) substitution will be fast.

17.(3) Theory based

18.(2) $\underset{\text{Hot and conc.}}{6NaOH} + Cl_2 \rightarrow NaClO_3 + 5NaCl + 3H_2O$

$2Ca(OH)_2 + Cl_2 \rightarrow Ca(OCl)_2 + CaCl_2 + H_2O$

19.(2)



20.(3) Vinyl halides and aryl halides do not give friedal craft reaction.

SECTION – 2

21.(–192.5)

$2C(s) + 3H_2(g) \rightarrow C_2H_6(g)$

$\Delta H_f^0(C_2H_6) = \Sigma \Delta H_C^0 \text{(reactants)} - \Sigma \Delta H_C^0 (\text{Products})$

$= 2 \times \Delta H_C^0 C(s) + 3\Delta H_C^0 H_2(g) - \Delta H_C^0 C_2H_6(g)$

$= 2 \times (-286) + 3 \times (-393.5) - (-1560)$

$= -192.5 \text{ kJ.}$

22.(18) $NaCl + k_2Cr_2O_7 + H_2SO_4 \rightarrow \underset{A}{CrO_2Cl_2} + \text{side products}$

$\underset{(A)}{CrO_2Cl_2} + NaOH \rightarrow \underset{(B)}{Na_2CrO_4} + \text{side products}$

$\underset{(B)}{Na_2Cr_2O_4} + H_2SO_4 + H_2O_2 \rightarrow \underset{(C)}{CrO_5} + \text{side products}$

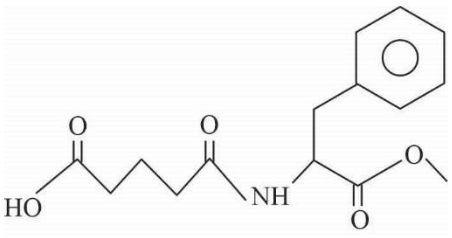
23.(0.37) m.moles of HCl required for 1 Litre $= 30$

m. moles of H_2SO_4 required for 1 Litre $= 15.$

m.moles of H_2SO_4 required for 250 mL $= \frac{15}{4}$

Weight of H_2SO_4 required $= \frac{15}{4} \times 10^{-3} \times 98 = 0.37g$

24.(9)



Aspartame

25.(5.23) m. moles of $CH_3COOH = 50$

m.moles of $HCl = 25$

m.moles of CH_3COOH in $20mL = \frac{50}{500} \times 20 = 2$

m. moles of HCl in $20mL = \frac{25}{500} \times 20 = 1$

Now

	$NaOH + HCl \rightarrow NaCl + H_2O$	
$t = 0$	2.5	1
t_{end}	1.5	0

	CH_3COOH +	$NaOH$ $\rightarrow$	$CH_3COONa + H_2O$
$t = 0$	2	1.5	0
t_{end}	0.5	0	1.5

In the end we have an acidic buffer

$$P_H = P_{Ka} + \log\frac{[Salt]}{[Acid]}$$

$P_H = 5.23$.

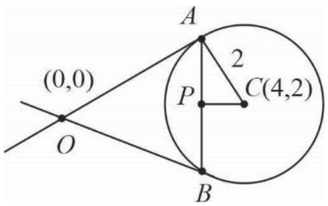
MATHEMATICS
SECTION – 1

1.(4) Let $z=\frac{3+i\sin\theta}{4-i\cos\theta}=\frac{(12-\cos\theta.\sin\theta)+i(3\cos\theta+4\sin\theta)}{16+\cos^2\theta}$

As z is real $\Rightarrow$ $3\cos\theta+4\sin\theta=0$

$\Rightarrow$ $\cot\theta=-\frac{4}{3}$ $\Rightarrow$ argument of $z=\pi-\tan^{-1}|\cot\theta|=\pi-\tan^{-1}\frac{4}{3}$ or $-\tan^{-1}\frac{4}{3}$

2.(2)



Equation of AB from $T=0$ is

$4x+2y=16\Rightarrow 2x+y=8$

$AB=2\sqrt{AC^2-CP^2}$

$CP=\left|\frac{2}{\sqrt{5}}\right|$ $\Rightarrow$ $AB=2\sqrt{4-\frac{4}{5}}=\frac{8}{\sqrt{5}}$

3.(2) According to $LMVT$

$f'(c)=\frac{f(b)-f(a)}{b-a}$

$3c^2-8c+3=0$

$c=\frac{8\pm\sqrt{28}}{6}=\frac{4\pm\sqrt{7}}{3}$

4.(2) The equation of Tangent is

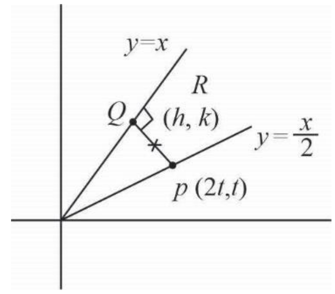
$\frac{x}{4\sqrt{2}}+\frac{y}{3\sqrt{2}}=1$ $\Rightarrow$ $a=4$ $\Rightarrow$ $2ae=2\sqrt{7}$

5.(3) $m_{PR}=-1$

$\Rightarrow$ $\frac{k-t}{h-2t}=-1$

$k-t=-h+2t$

$t=\frac{h+k}{3}$ ....(1)



The point Q will be $(2h-2t,2k-t)$

It will lie on $y=x$

$\Rightarrow$ $2h-2t=2k-t$

$2h = 2k + t$ ….(2) from (1) and (2)

$2h = 2k + \frac{h+k}{3}$

$6h = 6k + h + k$

$5x - 7y = 0$

6.(4) Coefficient of x^7 is $= {}^{10}C_7 + {}^{9}C_6 .... + {}^{3}C_0$

$= {}^{11}C_4 = 330$

7.(4) Let $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} 3^0 a_{11} & 3^1 a_{21} & 3^2 a_{31} \\ 3^1 a_{12} & 3^2 a_{22} & 3^3 a_{32} \\ 3^2 a_{13} & 3^3 a_{23} & 3^4 a_{33} \end{bmatrix}$

$|B| = 3 \times 3^2 \times 3 \times 3^2 |A| \quad \Rightarrow \quad |A| = \frac{1}{9}$

8.(3)

9.(1) $S = (3 + 8 + 13......) + (4 + 9 + 14 + ...)$

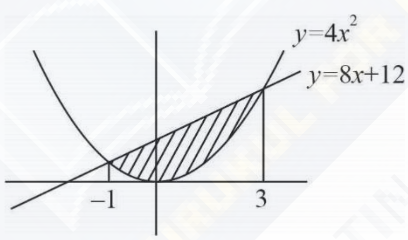
$\Rightarrow \quad S = \frac{20}{2}[6 + 19 \times 5] + \frac{20}{2}[8 + 19 \times 5] = 20 \times 102$

10.(3) $4\alpha\left(\int_{-1}^{0} e^{\alpha x} + \int_{0}^{2} e^{-2x}\right) = 5$

$4\left[1 - e^{-\alpha} - e^{-2\alpha} + 1\right] = 5 \qquad e^{-\alpha} + e^{-2\alpha} = \frac{3}{4}$

$3e^{2\alpha} - 4e^{\alpha} - 4 = 0 \quad \Rightarrow \quad e^{\alpha} = 2 \Rightarrow 2 = \ln 2$

11.(1)



$\Rightarrow \quad \int_{-1}^{3} (8x + 12) - 4x^2 = \frac{-4}{3}x^3 + 4x^2 + 12x|_{-1}^{3}$

$= \frac{128}{3}$

12.(2) We have $\alpha^2 = \alpha + 1 \Rightarrow \alpha^{k+2} = \alpha^{k+1} + \alpha^k$ …..(1)

Similarly $\beta^{k+2} = \beta^{k+1} + \beta^k$ …..(2)

$\Rightarrow \quad P_{k+2} = P_{k+1} + P_k$

$P_1 = 1, P_2 = 3, P_3 = 4, P_4 = 7, P_5 = 11$

13.(2) $P={}^5C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^5+{}^5C_1\left(\frac{1}{4}\right)\left(\frac{1}{3}\right)^4+{}^5C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$

$=\frac{3^3}{4^5}[9+15+10]=\frac{17}{8}$

14.(2) $6.{}^{35}C_r=\left(k^2-3\right){}^{36}C_{r+1}$

$k^2-3=6.\frac{{}^{35}C_r}{{}^{36}C_{r+1}}$

$k^2=\frac{6}{36}(r+1)+3$

$k^2=\frac{r+1}{6}+3$

For k to be an integer.

$r=5\Rightarrow k=\pm 2$

$r=35\Rightarrow k\pm 3.$ $\quad\therefore\quad$ 4 ordered pairs are possible

15.(4) $2\cot^2\theta-\frac{5}{\sin\theta}+4=0 \quad\Rightarrow\quad 2\left(1-\sin^2\theta\right)-5\sin\theta+4\sin^2\theta=0$

$\Rightarrow\quad 2\sin^2\theta-5\sin\theta+2=0 \quad\Rightarrow\quad \sin\theta=\frac{1}{2},\ \sin\theta=2$ (not possible)

$\theta=\frac{\pi}{6},\frac{5\pi}{6} \quad\Rightarrow\quad \theta_1=\pi/6,\quad \theta_2=5\pi/6$

$\Rightarrow\quad \int_{\pi/6}^{5\pi/6}\cos^2 3\theta d\theta=\int_{\pi/6}^{5\pi/6}\frac{1+\cos 6\theta}{2}d\theta$

$=\frac{1}{2}\theta+\frac{1}{12}\sin 6\theta\Big|_{\pi/6}^{5\pi/6}=\frac{\pi}{3}$

16.(2) $y\sqrt{1-x^2}=k-x\sqrt{1-y^2}$

Differentiating both sides wrt x.

$\frac{-yx}{\sqrt{1-x^2}}+\sqrt{1-x^2}\frac{dy}{dx}=-\sqrt{1-y^2}-\frac{xy}{\sqrt{1-y^2}}\frac{dy}{dx}$

Put $x=\frac{1}{2},y=\frac{-1}{4} \quad\Rightarrow\quad \frac{dy}{dx}=\frac{-\sqrt{5}}{2}.$

17.(4) $f(x)=ax^5+bx^4+cx^3$

Then $\lim_{x\to 0}\left(2+\frac{ax^5+bx^4+cx^3}{x^3}\right)=4$

$\Rightarrow\quad c+2=4$

$c=2$

Also $x = \pm 1$ are its critical pts.

$f'(x) = 5ax^4 + 4bx^3 + 3cx^2 \qquad = x^2\left(5ax^2 + 4bx + 3c\right)$

$f'(1) = 0 \quad \Rightarrow \quad 5a + 4b + 3c = 0$

$f'(-1) = 0 \quad \Rightarrow \quad 5a - 4b + 3c = 0$

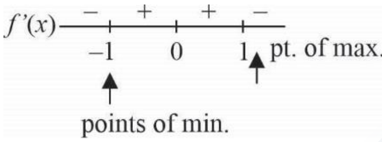
$5a + 4b + 6 = 0; \qquad 5a - 4b + 6 = 0$

$10a = -12 \qquad a = -6/5$ and $b = 0$

$\therefore \qquad f(x) = \frac{-6}{5}x^5 + 2x^3$

$f'(x) = -6x^4 + 6x^2$

$= -6x^2\left(x^2 - 1\right) \qquad = -6x^2(x-1)(x+1)$



18.(1) $a_1, a_2, a_3 .........G.P.$

$a_1 < 0$

$a_1 + a_2 = 4, \qquad a_3 + a_4 = 16.$

$a_1 + a_1 r = 4, \qquad a_1 r^2 + a_1 r^3 = 16$

$a_1(1+r) = 4, \qquad a_1 r^2 (1+r) = 16$

$\frac{a_1 r^2 (1+r)}{a_1(1+r)} = \frac{16}{4} \Rightarrow \qquad r^2 = 4$

$r = \pm 2$

$r = -2 \qquad a_1 = -4$

$\sum_{i=1}^{9} a_i = 4\lambda$

$a_1 + a_2 + ... + a_9 = 4\lambda$

$\frac{a_1\left(1-(r)^9\right)}{1-r} = 4\lambda$

$\frac{-4\left(1-(-2)^9\right)}{1-(-2)} = 4\lambda$

$-\frac{4}{3}(1+512) = 4\lambda$

$\lambda = \frac{-513}{3} \qquad \lambda = -171$

19.(1) $\left(y^2 - x\right)\frac{dy}{dx} = 1$

$\frac{dx}{dy} + x = y^2$

$I.F. \quad e^{\int dy} = e^y$

Solution is $xe^y = \int y^2 e^y dy + c$

$xe^y = y^2 e^y - 2ye^y + 2e^y + c$

$x = y^2 - 2y + 2 + ce^{-y}$

But $y(0) = 1$

$c = -e$

At $y = 0 \qquad x = 2 - e.$

20.(3) $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$\vec{a} + \vec{b} + \vec{c} = \vec{0} \quad \Rightarrow \quad |\vec{a} + \vec{b} + \vec{c}| = 0$

$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\left(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}\right) = 0$

$3 + 2\lambda = 0 \qquad \lambda = -3/2.$

$\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} \qquad = \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{a}$

$\vec{d} = 3\left(\vec{a} \times \vec{b}\right)$

SECTION – 2

21.(13) $x + y + z = 6$

$x + 2y + 3z = 10$

$3x + 2y + \lambda z = \mu$

In first two equations. Put $z = 0$

$x + y = 6$

$x + 2y = 10 \qquad y = 4, \quad x = 2$

$\therefore \quad (2, 4, 0)$ should satisfy plane (3)

$6 + 8 = \mu \qquad \mu = 14$

Now put $y = 0$

$x + z = 6$

$x + 3z = 10$

$2z = 4 \quad \Rightarrow \quad z = 2$

$\therefore \quad (4, 0, 2)$ is a point on plane (3)

$12 + 2\lambda = 14$

$2\lambda = 2 \qquad \lambda = 1.$

$\therefore \quad \mu - \lambda^2 = 14 - 1 = 13.$

22.(54) Observations are

$3, 7, 9, 12, 13, 20, x, y$

$\sigma^2 = 25$ and $\bar{x} = 10$

$\Rightarrow \quad \dfrac{3+7+9+12+13+20+x+y}{8} = 10$

$x + y = 80 - 64$

$x + y = 16$

$\sigma^2 = \dfrac{\sum x_i^2}{n} - \bar{x}^2$

$25 = \dfrac{3^2+7^2+9^2+12^2+13^2+20^2+x^2+y^2}{8} - (10)^2$

$25 \times 8 = 852 + x^2 + y^2 - 800$

$200 = x^2 + y^2 + 52$

$x^2 + y^2 = 148$

$(x+y)^2 - 2xy = 148$

$256 - 2xy = 148$

$xy = 54$

23.(5) $f(x) \begin{cases} \dfrac{1}{x}\log_e\left(\dfrac{1+3x}{1-2x}\right), & x \neq 0 \\ k, & x = 0 \end{cases}$

$f(x)$ is continuous at $x = 0$

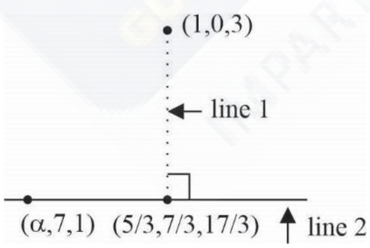
$\therefore \quad \lim_{x\to 0} f(x) = f(0)$

$\Rightarrow \quad k = \lim_{x\to 0} \dfrac{1}{x}\left(\log(1+3x) - \log(1-2x)\right)$

$= \lim_{x\to 0} \dfrac{\log(1+3x)}{x} - \dfrac{\log(1-2x)}{x}$

$= 3 + 2 = 5$.

24.(4)



D.R's of line 1

$= \left(\dfrac{5}{3}-1\right), \left(\dfrac{7}{3}-0\right), \left(\dfrac{17}{3}-3\right)$

$= \frac{2}{3}, \frac{7}{3}, \frac{8}{3} \qquad = 2, 7, 8$.

D.R.'s of line 2

$= \left(\alpha - \frac{5}{3}\right), \left(7 - \frac{7}{3}\right), \left(1 - \frac{17}{3}\right) \qquad = \left(\alpha - \frac{5}{3}\right), \frac{14}{3}, \frac{-14}{3}$

$= \frac{3\alpha - 5}{3}, \frac{14}{3}, \frac{-14}{3}. \qquad = 3\alpha - 5, 14, -14$

Line 1 is $\perp$ to line 2.

$\Rightarrow \quad 2(3\alpha - 5) + 7 \times 14 + 8 \times (-14) = 0$

$\Rightarrow \quad 6\alpha - 10 + 14(7 - 8) = 0$

$\Rightarrow \quad 6\alpha - 10 - 14 = 0$

$\alpha = 4$

25.(29) $X = \{1, 2, 3, ..., 50\}$

$A = \{2, 4, 6, 8, ..., 50\}$

$n(A) = 25$

$B = \{7, 14, 21, ..., 49\}$

$n(B) = 7, \qquad A \cap B = \{14, 28, 42\}$

$n(A \cap B) = 3$

Now smallest subset of X containing both A and B is $= A \cup B$

$\Rightarrow \quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 25 + 7 - 3$

$= 29$