Vidyamandir Classes
Gurukul for IITJEE & Medical Preparation

SOLUTIONS

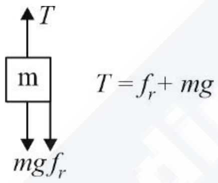
JEE Main – 2020 | 7th January 2020 (Morning)

PHYSICS

SECTION – 1

1.(2) A logic gate is reversible if we can recover input data from the output.

2.(1) $m = 2000\,\text{kg}$
$f_r = 4000\,N$
$v = ?$
$T = 4000 + 20000$, $T = 24000N$
$P = T.v$;
$60 \times 746 = 24000 \times u$, $u = 1.9$ m/s



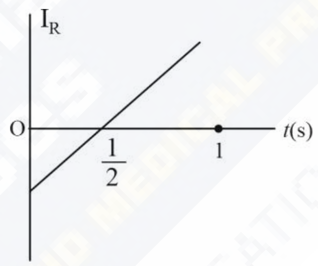
3.(1) $I = I_0 t - I_0 t^2$

$\phi = BA = \mu_0 n \text{IA}$

$V_R = -\dfrac{d\phi}{dt} = -\mu_0 n A I_0 (1-2t)$

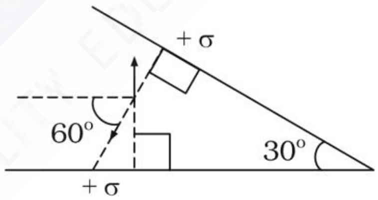
$V_R = 0$ at $t = \dfrac{1}{2}$ sec

$I_R = \dfrac{V_R}{2R_0} = \dfrac{-2\mu_0 n I_0 \pi R^2}{R_0}(1-2t)$



4.(4) $\vec{E} = \dfrac{-\sigma}{2\epsilon_0}\cos 60^\circ \hat{x} + \left(\dfrac{\sigma}{2\epsilon_0} - \dfrac{\sigma}{2\epsilon_0}\sin 60^\circ\right)\hat{y}$

$= \dfrac{\sigma}{2\epsilon_0}\left[\left(1-\dfrac{\sqrt{3}}{2}\right)\hat{y} - \dfrac{1}{2}\hat{x}\right]$



5.(4) $\lambda = 6000 \times 10^{-8}$ cm

For 2nd minimum $d\sin\theta_2 = 2\lambda \quad \Rightarrow \quad \dfrac{\lambda}{d} = \dfrac{\sqrt{3}}{4}$

So, for 1st minimum, $d\sin\theta_1 = \lambda \quad \Rightarrow \quad \sin\theta_1 = \dfrac{\lambda}{d} = \dfrac{\sqrt{3}}{4}$

$\therefore \quad \theta_1 = 25.65^\circ$ (from sin table), $\theta_1 \approx 25^\circ$

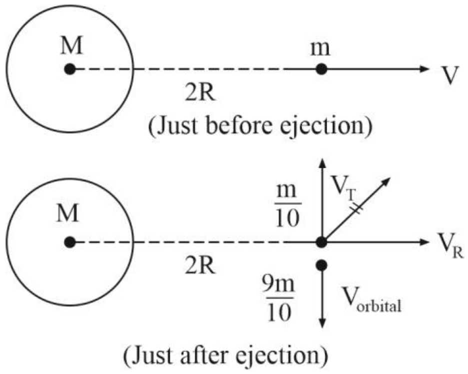
6.(1) Energy conservation :

$-\dfrac{GMm}{R} + \dfrac{1}{2}mu^2 = \dfrac{-GMm}{2R} + \dfrac{1}{2}mv^2$

$V = \sqrt{u^2 - \dfrac{GM}{R}}$ …(i)

Momentum conservation:

$$\frac{m}{10}V_T = \frac{9m}{10}\sqrt{\frac{GM}{2R}}\left[V_{orbital} = \sqrt{\frac{GM}{2R}}\right]$$



$$\frac{m}{10}V_r = m\sqrt{u^2 - \frac{GM}{R}} \quad \text{[By (1)]}$$

Kinetic energy $= \frac{1}{2} \cdot \frac{m}{10}\left(V_T^2 + V_r^2\right) = \frac{m}{20}\left(\frac{81GM}{2R} + 100u^2 - 100\frac{GM}{R}\right) = (1)$

7.(4) $\gamma_{\text{mix}} = \frac{C_{p\,\text{mix}}}{C_{V\,\text{mix}}} = \frac{n_1C_{P_1} + n_2C_{P_2}}{n_1C_{V_1} + n_2C_{V_2}} \Rightarrow \gamma_{\text{mix}} = \dfrac{n_1\frac{\gamma_1 R}{\gamma_1 - 1} + n_2\frac{\gamma_2 R}{\gamma_2 - 1}}{\frac{n_1R_1}{\gamma_1 - 1} + \frac{n_2 R}{\gamma_2 - 1}}$

On rearranging, we get, $\frac{n_1 + n_2}{\gamma_{\text{mix}} - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$

$\Rightarrow \quad \frac{5}{\gamma_{\text{mix}} - 1} = \frac{3}{1/3} + \frac{2}{2/3} \quad \Rightarrow \quad \gamma_{\text{mix}} - 1 = \frac{5}{12} \Rightarrow \gamma_{\text{mix}} = \frac{17}{12} = 1.42$

8.(1) Time-period $= T = \frac{2\pi r}{v}$

$$T = \frac{2\pi a_0}{V_1} \cdot \frac{n^2}{Z} \cdot \frac{n}{Z} \;;\; T = \frac{2\pi a_0}{V_1} \cdot \frac{n^3}{Z^2}$$

$$T \propto n^3$$

$\therefore \quad \frac{T_1}{T_2} = \frac{n_1^3}{n_2^3} = \frac{1}{8} \Rightarrow T_2 = 8T_1 \;;\; T_2 = 12.8 \times 10^{-16}\text{ sec}$

$\therefore \quad f_2 = \frac{1}{12.8 \times 10^{-16}} \approx 7.8 \times 10^{14}$

9.(2) Loss in P.E. = Gain in K.E.

$\Rightarrow \quad mgh = \frac{1}{2}mv^2 + \frac{1}{2}Iw^2$

Put, $v = wr$ (no slipping)

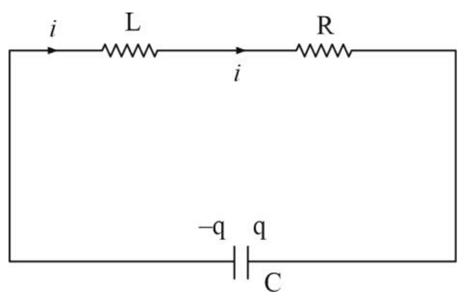
$\Rightarrow \quad mgh = \frac{1}{2}mw^2r^2 + \frac{1}{2} \cdot \frac{mr^2}{2} \cdot w^2 \quad \Rightarrow \quad mgh = \frac{3}{4}mw^2r^2 \quad \therefore\; w = \frac{1}{r}\sqrt{\frac{4gh}{3}}$

10.(3) Equal number of magnetic field lines enters the circular region & comes out of infinite plane excluding circular area. So, magnetic flux are equal in magnitude but opposite in direction.

$\phi_i = -\phi_0$

11.(1) For damped oscillation : $ma + bv + kx = 0$

$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \qquad \ldots\text{(i)}$



For LCR series circuit $-iR - L\frac{di}{dt} - \frac{q}{C} = 0$

$\Rightarrow \quad L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0 \qquad \ldots\text{(ii)}$

Comparing equation (i) and (ii), $L \leftrightarrow m,\; C \leftrightarrow \frac{1}{k},\; R \leftrightarrow b$

12.(4) $m = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)$, if final image is least distance of distinct vision

$\Rightarrow \quad 375 = \frac{150}{5}\left(1 + \frac{25}{f_e}\right)$

$$f_e = \frac{750}{345} = 2.17 \text{ cm} = 21.7 \text{ mm } \approx 22 \text{ mm}$$

Also, $m = \frac{L}{f_0}\left(\frac{D}{f_e}\right)$ if final image is at infinity.

$\Rightarrow \quad 375 = \frac{150}{5}\left(\frac{25}{f_e}\right), \qquad f_e = 22 \text{ mm}$

13.(4) $E_0 = B_0 C, C =$ speed of light in vacuum

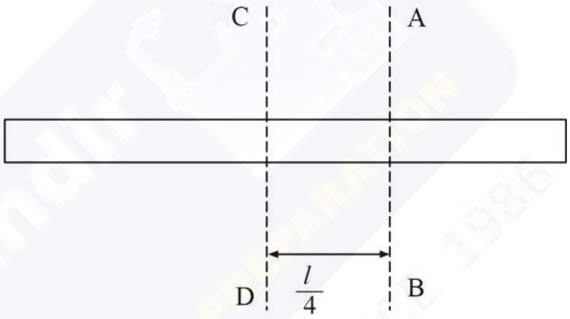
$E_0 = 3 \times 10^{-8} \times 3 \times 10^{8}$ V/m

$E_0 = 9$ V/m $\quad \therefore \quad \vec{E} = 9 \sin(1.6 \times 10^3 x + 48 \times 10^{10} t)\hat{k}$ V/m

14.(1) $I_{AB} = I_{CD} + md^2$

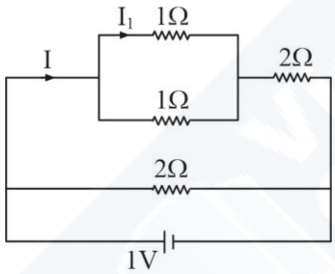
$= \frac{ml^2}{12} + \frac{ml^2}{16}$

$= \frac{7ml^2}{48}$



Radius of gyration $= \sqrt{\frac{I_{AB}}{m}} = \sqrt{\frac{7l^2}{48}} = \sqrt{\frac{7}{48}}l$

15.(4) $I = \frac{1}{2.5}A = 0.4A$



$I_1 = \frac{I}{2} = 0.2\,A$

16.(4) Take origin at 1 kg mass y = vertical, x = horizontal

$X\text{cm} = \frac{1 \times 0 \times 1.5 \times 3 + 2.5 \times 0}{5} = 0.9$ cm ; $Ycm = \frac{1 \times 0 + 1.5 \times 0 + 2.5 \times 4}{5} = 2$ cm

17.(1) $I = I_0 \cos^2\theta, \quad \frac{I_0}{10} = I_0 \cos^2\theta$

$\cos\theta = \frac{1}{\sqrt{10}} = 0.31 < 0.707$

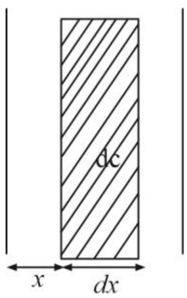
$\therefore \quad \theta > 45° \;\&\; 90° - \theta < 45°$; $\theta = 71.6°$ $\quad \therefore$ Angle rotated $= 90° - 71.6° = 18.4°$

18.(4) All $dc's$ are in series

$\therefore \quad \frac{1}{c} = \frac{1}{dc} + \frac{1}{dc} + \frac{1}{dc} + ...$

$\frac{1}{c} = \int \frac{1}{dc}$

$dc = \frac{k \in_0 A}{dx}$



$$\frac{1}{c}=d\int_0^d \frac{dx}{(1+\alpha x)\in_0 AK}=\frac{1}{K\in_0 A\alpha}\ln|1+\alpha d|$$

$\alpha d << 1$

$$\therefore \ \ln|1+\alpha d|=\alpha d-\frac{\alpha^2 d}{2}$$

$$\frac{1}{c}=\frac{\alpha d}{K\in_0 A\alpha}\left(1-\frac{\alpha d}{2}\right) \ ; \ c=\frac{K\in_0 A}{d}\left(1+\frac{\alpha d}{2}\right)$$

19.(1) $m=6.0\,g$, $l=60\text{ cm}$, $A=1.0\text{ mm}^2$, $v=90\text{ ms}^{-1}$, $Y=16\times10^{11}\text{ Nm}^{-2}$

$$v=\sqrt{\frac{T}{\mu}} \ \therefore T=\mu v^2 \ ; \ \frac{\mu v^2}{A}=Y\frac{\Delta l}{l}$$

$$\Delta l=\frac{\mu v^2 l}{AY}=\frac{mv^2}{AY}=\frac{6\times10^{-3}\times8100}{10^{-6}\times16\times10^{11}}m=3\times10^{-5}\text{ m}=0.03\times10^{-3}\text{ m}=0.03\text{ mm}$$

20.(1) $P_1V_1=P_2V_2 \Rightarrow P_2=P_1\left(\frac{V_1}{V_2}\right)^{\gamma}=1atm\left(\frac{1}{3}\right)^{1.4}$

$$\text{Work}=\frac{P_1V_1-P_2V_2}{\gamma-1}=\frac{10^{-3}\times1.01\times10^5\left(1-\frac{3}{4.65}\right)}{0.4}J$$

Work $\approx 90J$

$\therefore$ Closest answer is 90.5 J

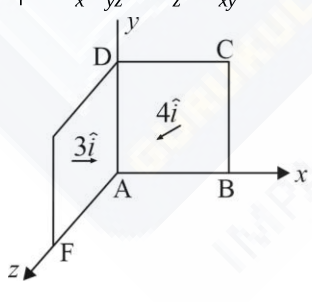
SECTION – 2

21.(600) $\eta=\frac{W}{Q_{in}}=1-\frac{300}{900}=\frac{2}{3}$; $Q_{in}=\frac{3}{2}W=1800J$; $Q_{low}=Q_{in}-Q=600\,J$

22.(60) $\gamma=2\alpha_2+\alpha_1=2\times5\times10^{-6}+5\times10^{-5}=60\times10^{-6}$ $\therefore$ $C=60$

23.(10) $KE=PE_A-PE_P=mgh_A-mgh_B=mg(2-1)=1\times10\times1=10\,J$

24.(175) $\phi=B_xA_{yz}+B_z\cdot A_{xy}=3\times25+4\times25=175\,Wb$



25.(11) $E_{photon}=\frac{1240}{310}=4ev$

No. of electrons emitted $=\frac{I.A.}{E_{photon}}n$

$$=\frac{6.4\times10^{-5}\times1}{4\times1.6\times10^{-19}}\times10^{-3}\times1=10^{11}$$

$\therefore \ x=11$

CHEMISTRY

SECTION – 1

1.(2) It is Gay Lussac law of gaseous volume

2.(4) $CS_2 + CH_3COCH_3$

A B

A……..A (non polar –non polar)

B……..B (polar –polar)

A……B (non polar…..polar)

So $\left.\begin{matrix}[A......A] \\ [B......B]\end{matrix}\right] > A......B$

So, it is non ideal solution showing positive deviation.

So, volume should be greater than 200ml

3.(1) $E°Cu^{++} / Cu = 0.34V \quad E°Cu^{+} / Cu = 0.522V$

$Cu^{++} + 2e \to Cu \qquad E° = 0.34V \qquad -(1) \qquad E^o_{Cu} ++ / Cu^+ = ?$

$Cu^{+}e \to Cu \qquad E° = 0.522V \qquad -(2)$

$Cu^{++} + e \to Cu^{+} \qquad E^o = ?$

(1)-(2) $\Rightarrow$

$Cu^{++} + 2e \to Cu \qquad E° = 0.34V \qquad -(3)$

$Cu \to Cu^{++} + e \qquad E^o = -0.522V \qquad -(4)$

$Cu^{++} + e \to Cu^{+} \qquad E° = ? \qquad -(5)$

$\Delta G_3 + \Delta G_4 = \Delta G_5$

$\Delta G = -nfE$

$\Delta G_3 = -2 \times F \times .34 \qquad \Delta G_4 = -1 \times F \times (-522)$

$\Delta G_4 = -1 \times f \times E°$

$(-2f \times .34) + [-1 \times (-.522)] = (-1 \times f \times E°)$

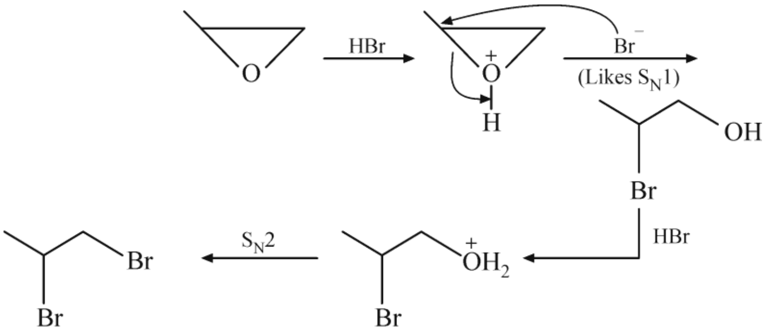
$E° = 0.158V$

4.(2)

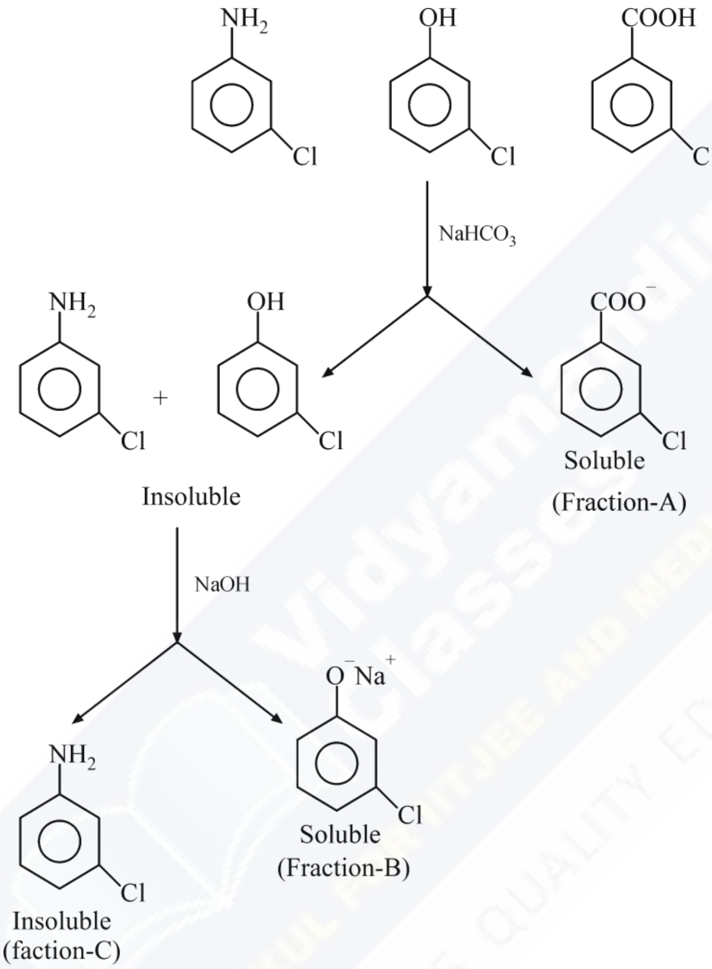
Vitamins	Deficiency
Vitamin B_2-(Riboflavin)	Cheilosis
Vitamin B_1 - (Thiamine)	Beriberi
Vitamin B_6 - (Pyridoxine)	Convulsions
Vitamin C - (Ascorbic acid)	scurvy)

5.(2) Diammine chloride (methanamine) (Platinum II) chloride

6.(2)



7.(1)



8.(4) $n = 5 \quad Ss, Sp, Sd, Sf, Sg$

$n^2 = 5^2 = 25$

No. of orbitals $= 1 + 3 + 5 + 7 + 9 = 25$

Each orbital has one electron with $m_5 = +\frac{1}{2}$

9.(4) Potassium is alkali metal

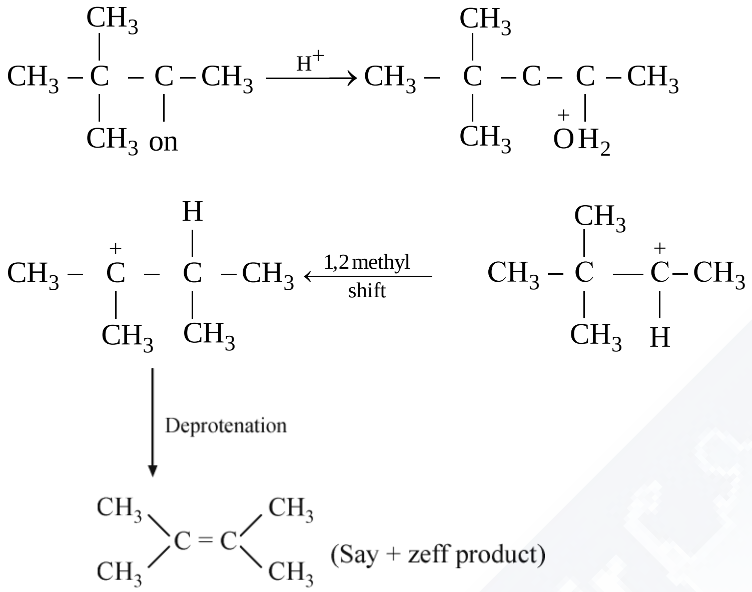
It always show +1 oxidation state

$K_2O \longrightarrow 2K^+ + O^{2-}$

$K_2O_2 \longrightarrow 2K^+ + O_2^{2-}$

$KO_2 \longrightarrow K^+ + O_2^-$

10.(3) (A)



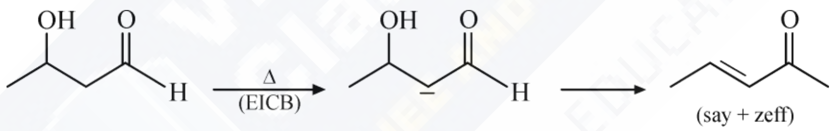
(B) $CH_3-CH(CH_3)-CH(Br)-CH_3 \xrightarrow[(E_2)]{\text{alcohol}} CH_3-C(CH_3)=C(H)-CH_3$

(C) $CH_3-CH(CH_3)-CH(Br)-CH_3 \xrightarrow{(CH_3)_3O^-K^+} CH_3-CH(CH_3)-CH=CH$

$(CH_3)_3O^-K^+ \longrightarrow$ Sterically hindered base (Hoffmann elimination)

(give Hoffmann major product)

(D)



11.(1) Due to lanthenoid contraction, size of $4d \simeq$ size of 5d series except La

12.(2) Fact

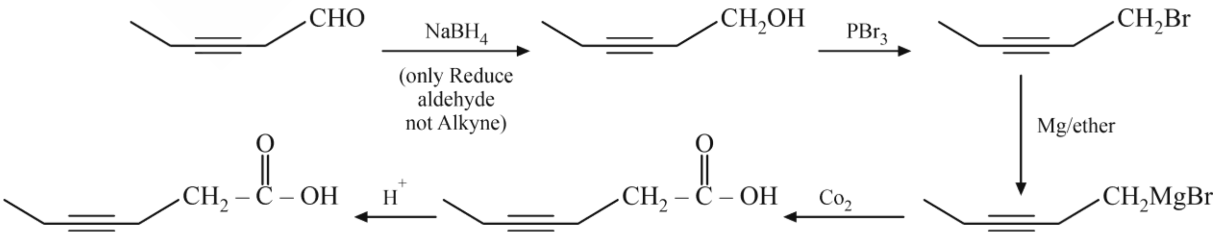
13.(2) Fact

14.(4) Dipole moment of CH_4 & $CC\ell4 = 0$

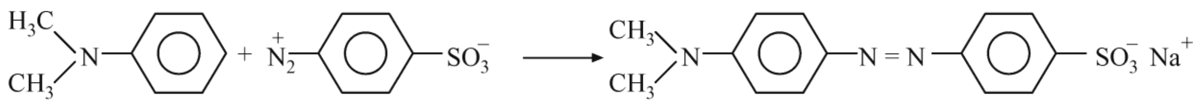
Dipole moment of $CHc\ell_3 \neq 0$

15.(4) $(c\ell > f > Br > I)$

16.(2)



17.(2) Fact

18.(2) 

Methyl orange
use in Acid Base Titration

19.(4) B $\rightarrow$ Guanidine Type strongest organic base (conjugate acid is stablised by equivalent Resonance) (three resonating structure)

A $\rightarrow$ $NH_2 - CH = NH$ (also guanidine type but two equivalent resonating structure of conjugate acid) (so less base then (B)

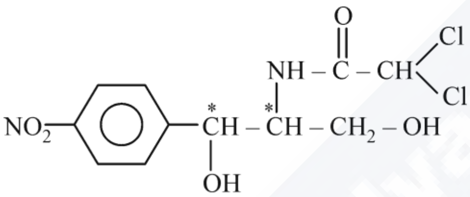
C $\rightarrow$ $CH_3\ NHCH_3$ (Aliphatic amine)

Order of basic nature $B > A > C$

Order of P^{k_b} $B < A < C$

20.(4) MOT can explain the nature of bonding due to synergic bonding $[Ni(CO)_4]$.

SECTION – 2

21.(2) 

(Two Chiral Centre)

22.(10.60)

$NaOH \ + \ H_2SO_4$
(A) (B)

Molarity of $NaOH = \dfrac{4}{40 \times 100} = 10^{-3}$

Molarity of $H_2SO_4 = \dfrac{9.8}{98 \times 100} = 10^{-3}$

$2NaOH \quad + \quad H_2SO_4 \rightarrow Na_2SO_4 + H_2O$

$10^{-3}M \qquad 10^{-3}M$

$40\ell \qquad 10\ell$

Moles $= 40 \times 10^{-3}$ moles $= 10 \times 10^{-3}$

Limiting Reagent is H_2SO_4

So, moles of NaOH left $= \left(40 \times 10^{-3}\right) - \left(10 \times 10^{-3} \times 2\right)$

20×10^{-3}

Molarity of $\left[OH^-\right] = \dfrac{20 \times 10^{-3}}{50} = 4 \times 10^{-4}$

$P^{OH} = 4 - \log 4 = 3.398$

$P^{H} = 14 - 3.398 = 10.602$

23.(23.03)

$t_{y_2} = 6.93 \text{years}$

$t = \dfrac{2.303}{K} \log \dfrac{100}{10}$

$K = \dfrac{0.693}{6.93} = 0.1$

$t = \dfrac{2.303}{.1} \times 1 = 23.03 \text{ years}$

24.(–2.70) $A_{(\ell)} \rightarrow 2B_{(g)}$

$\Delta U = 2.1 k\,cal, \Delta s = 20 cal/k,$ $T = 300K$

$\Delta G = \Delta H - T\Delta S$

$\Delta H = \Delta U + \Delta ng\,RT$ $\Delta ng = 2$

$\Delta g = 3300 - (300)(20)$

$= 3300 - 6000$

$= -2700 cal$

$= -2.7 kcal$

25.(1.67) $3Cl_2 + \underset{(Hot \& conc)}{6NaOH} \rightarrow \underset{(x)}{5NaCl} + \underset{(y)}{NaClO_3} + 3H_2O$

$NaCl + AgNO_3 \rightarrow AgCl \downarrow NaNO_3$

Y is N $NaClO_3$ $Na^+ ClO_3^-$

$^-O - \ddot{Cl} = 0$
$\quad\quad ||$
$\quad\quad O$

Bond order =
(in Resonance)

$= 1 + \dfrac{\text{no of } \pi \text{ band}}{\text{no.of } \sigma \text{ bond}}$

$= 1 + \dfrac{2}{3}$

$= \dfrac{5}{3} = 1.67$

MATHEMATICS

SECTION – 1

1.(1) $\frac{dy}{dx}-1=e^x\cdot e^{-y}$

$e^y\frac{dy}{dx}-e^y=e^x$

$e^y\cdot\frac{dy}{dx}=e^x+e^y$

$e^y=t$

$e^y\frac{dy}{dx}=\frac{dt}{dt}$

$\frac{dt}{dx}=e^x+t$

$\frac{dt}{dx}-t=e^x$

I.F. $e^{-\int 1dx}=e^{-x}$

$t\cdot e^{-x}=\int e^x\cdot e^{-x}dx$

$t\cdot e^{-x}=x+c$

$t=xe^x+ce^x$

$e^y=xe^x+ce^x$

$x=0,\ y=0$

$1=0+c \quad\Rightarrow\quad c=1$

At $x=1$

$e^y=e^x(x+1)$

$e^y=e(2)$

$\log_e e^y=\log_e 2e$

$y=\log_e 2+1$

2.(4) Given that plane passes through the point $\frac{(2,1,0)}{x_1,y_1,z_1}$, $\frac{(4,1,1)}{x_2,y_2,z_2}$ and $\frac{(5,0,1)}{x_3,y_3,z_3}$.

Equation of plane passes through 3 non-collinear point

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

$$\begin{vmatrix} x-2 & y-1 & z-0 \\ 4-2 & 1-1 & 1-0 \\ 5-2 & 0-1 & 1-0 \end{vmatrix}=0$$

$$\begin{vmatrix} x-2 & y-1 & z \\ 2 & 0 & 1 \\ 3 & -1 & 1 \end{vmatrix}=0$$

$\Rightarrow \quad (x-2)(0+1)-(y-1)(-1)+z(-2)=0 \quad \Rightarrow \quad x-2+y-1-2z=0$

$x-y-2z=3$

Now image of $R(2,1,6)$ w.r.t $x+y-2z=3$ is

$$\frac{x_2-2}{1}=\frac{y_2-1}{1}=\frac{z_2-6}{-x}=\frac{-2(2+1-12-3)}{6}$$

$$\frac{x_2-2}{1}=\frac{y_2-1}{1}=\frac{z_2-6}{-2}=4$$

$x_2=6 \quad y_2=5 \quad z_2=-2$

3.(4) Let *x* be a random variable and *k* be the value of assigned *x* for $k=3,4,5$

k	0	1	2	3	4	5
$P(k)$	$\frac{1}{32}$	$\frac{12}{32}$	$\frac{11}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{1}{32}$

Now expected value $=\Sigma xP(k)$

$$=-1\times\frac{1}{32}+(-1)\times\frac{12}{32}+(-1)\times\frac{11}{32}+3\times\frac{5}{32}+4\times\frac{2}{32}+5\times\frac{1}{32} \quad \Rightarrow \quad \frac{-24+28}{32}=\frac{4}{32}=\frac{1}{8}$$

4.(4) $y=mx+4$

Let $y=mx+\frac{1}{m}$ is tangent to $y^2=4x$

Now solving with $x^2=2by$

$$x^2=2b\left(mx+\frac{1}{m}\right)$$

$$x^2-2bmx-\frac{2b}{m}=0$$

$x=0$ (because line touches the curve)

$$(-2bm)^2-4\times1\times\left(-\frac{2b}{m}\right)=0$$

$$4b^2m^2+\frac{8b}{m}=0 \quad \Rightarrow \quad 4b^2m^2=-\frac{8b}{m}$$

$$bm^3=-2 \quad \Rightarrow \quad m^3=-\frac{2}{b} \quad \Rightarrow \quad m=-\left(\frac{2}{b}\right)^{1/3}$$

Now $\quad -\left(\frac{b}{2}\right)^{1/3}=4$

$$-\left(\frac{b}{2}\right)=64 \quad \Rightarrow \quad b=-128$$

2nd method

Given $\quad y=mx+4 \qquad$ .... (i)

Let the equation of common tangent of $y^2=4x$ & $x^2=2by$ is

$$y=mx+\frac{1}{m} \qquad \text{........ (ii)}$$

(i) & (ii) are identical

$$m=\frac{1}{4}$$

So the line $y=\frac{1}{4}x+4$ is also common tangent of $x^2=2by$

Solving $x^2=2b\left(\frac{x+16}{4}\right)$

$$2x^2=bx+16b$$

$$2x^2-bx-16b=0$$

$\Rightarrow$ $D=0$

$$b^2-4\times2\times(-16b)=0$$

$$b^2+128b=0$$

$b=-128$ & $b=0$ (which is not possible)

So $b=-128$

5.(2) $49^{125}+49^{124}+.....+49^2+49+1$

Clearly given series in G.P.

$$\Rightarrow \quad \frac{1(49^{126}-1)}{49-1}=\frac{49^{126}-1}{48}=\frac{(49^{63}+1)(49^{63}-1)}{49}$$

Greatest value of $k=63$

6.(4) Given digits are 1, 3, 5, 7, 9

For digits to repeat we have 5 choices, hence total members are, ${}^5C_1\times\frac{6!}{2!}=\frac{5\times6!}{2}$

7.(3) Given equation, $x^k+y^k=a^k$

Differentiating w.r.t. x,

$$kx^{k-1}+ky^{k-1}\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-\left(\frac{x}{y}\right)^{k-1} \quad .... \text{(i)}$$

& given that

$$\frac{dy}{dx}=-\left(\frac{y}{x}\right)^{1/3} \quad .... \text{(ii)}$$

(i) & (ii) are tangent

$$\left(\frac{x}{y}\right)^{k-1}=\left(\frac{x}{y}\right)^{-1/3} \Rightarrow k-1=-\frac{1}{3} \Rightarrow k=\frac{2}{3}$$

8.(3) Given equation $(k+1)\tan^2 x-\sqrt{2}\lambda\tan x=1-k$

$$(k+1)\tan^2 x-\sqrt{2}\lambda\tan x+(k-1)=0$$

Sum of roots $\tan\alpha+\tan\beta=-\frac{b}{a}=\frac{\sqrt{2}\lambda}{k+1}$

Product of roots $\tan\alpha\tan\beta=\frac{c}{a}=\frac{k-1}{k+1}$

$$\tan^2(\alpha+\beta) = (\tan(\alpha+\beta))^2 = 50 = \left(\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}\right) = 50 = \left(\frac{\frac{\sqrt{2}\lambda}{k+1}}{1-\frac{k-1}{k+1}}\right)^2 = 50$$

$$\Rightarrow \quad \left(\frac{\frac{\sqrt{2}\lambda}{k+1}}{\frac{k+1-k+1}{k+1}}\right)^2 = 50 \quad \Rightarrow \quad \left(\frac{\lambda}{\sqrt{2}}\right)^2 = 50 \quad \Rightarrow \quad \lambda^2 = 100 \quad \Rightarrow \quad \lambda = \pm 10$$

9.(2) Given equation,

$$x^2 + x + 1 = 0$$

$$\alpha = \omega, \omega^2$$

Taking $\alpha = \omega$

$$A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{bmatrix}$$

$$A^2 = \begin{pmatrix} 1+1+1 & 1+\omega+\omega^2 & 1+\omega^2+\omega \\ 1+\omega+\omega^2 & 1+\omega^2+\omega^4 & 1+\omega^4+\omega^2 \\ 1+\omega^2+\omega & 1+\omega^3+\omega^3 & 1+\omega^4+\omega^3 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 3 & 0 \end{pmatrix}$$

$$\Rightarrow \quad A^2 = I \quad \Rightarrow \quad A^4 = I$$

Now, $A^{31} = A^{28} \times A^3 = (A^4)^7 \times A^3 \quad \Rightarrow \quad I \times A^3 = A^3$

10.(3) The system of linear equations

$$2x + 2ay + az = 0$$

$$2x + 3by + bz = 0$$

$$2x + 4cy + cz = 0$$ has non zero solution

Then $D = 0$,

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

$$1(bc - 4bc) - 2a(c-b) + a(4c - 3b) = 0$$

$$\Rightarrow \quad 3bc - 4bc - 2ac + 2ab + 4ac - 3ab = 0$$

$$\Rightarrow \quad -bc + 2ac + 2ab - 3ab = 0$$

$$-bc - ab = -2ac$$

$$bc + ab = 2ac$$

$$\Rightarrow \quad \frac{2}{b} = \frac{1}{a} + \frac{1}{c} \quad \Rightarrow \quad \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \text{ are in AP}$$

11.(2) Given statement

$(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$

p	q	$p \Rightarrow q$	$\sim p$	$q \Rightarrow \sim p$	$(p \Rightarrow q)(p \Rightarrow -q)$
T	T	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

$\Rightarrow \quad (p \Rightarrow q) \wedge (p \Rightarrow \sim q) = \sim p$

12.(3) $y(x) = \sqrt{2\left(\frac{\tan\alpha + \cot\alpha}{1+\tan\alpha}\right) + \frac{1}{\sin^2 x}}$

$$\Rightarrow \quad \sqrt{2\left(\frac{\frac{\sin x}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}}{1+\frac{\sin^2\alpha}{\cos^2\alpha}}\right) + \frac{1}{\sin^2\alpha}} \quad \Rightarrow \quad \sqrt{2\times\frac{\left(\frac{\cos^2\alpha + \sin^2\alpha}{\sin\alpha\cos\alpha}\right)}{\frac{\cos^2\alpha + \sin^2\alpha}{\cos^2\alpha}} + \frac{1}{\sin^2\alpha}}$$

$$\Rightarrow \quad \sqrt{2\times\frac{\cos^2\alpha}{\sin\alpha\cos\alpha} + \frac{1}{\sin^2 x}} \quad \Rightarrow \quad \sqrt{2\cot\alpha + \operatorname{cosec}^2\alpha}$$

$$\Rightarrow \quad \sqrt{1+\cot^2\alpha + 2\cot\alpha} \quad \Rightarrow \quad |1+\cot\alpha| \quad \Rightarrow \quad -(1+\cot\alpha)$$

for $\alpha \in \left(\frac{3\pi}{4}, \pi\right)$

$$\frac{dy}{d\alpha} = \operatorname{cosec}^2\alpha$$

$$\frac{dy}{d\alpha} \text{ at } \alpha = \frac{5\pi}{6} = \operatorname{cosec}^2\frac{5\pi}{6} = 4$$

13.(3) $z = x + iy$

$$\frac{z-1}{2z+i} = \frac{(x-1)+iy}{2x+(2y+1)i} = (x-1)+iy\times(2x-(2y+1)i)$$

$$= (2x+(2y+1)i)\times(2x-(2y+1)i)$$

$$\text{Re}(z) = \frac{2x^2+2x+2y^2+y}{4x^2+4y^2+4y+1} = 1$$

$$2x^2 - 2x + 2y^2 + y = 4x^2 + 4y^2 + 4y + 1$$

$$\Rightarrow \quad 2x^2 + 2y^2 + 2x + 3y + 1 = 0$$

$$x^2 + y^2 + x + \frac{3}{2}y + \frac{1}{2} = 0$$

$$r = \sqrt{g^2 + f^2 - c} = \sqrt{\frac{1}{4} + \frac{9}{16} - \frac{1}{2}} = \frac{\sqrt{5}}{4}$$

$\therefore$ Circle whose diameter is $\frac{\sqrt{5}}{2}$.

14.(1) Using LMVT for $x \in [-7, -1]$

$$\frac{f(-1)-f(-7)}{(-1+7)} \le 2$$

$$\frac{f(-1)+3}{6} \le 2$$

$$f(-1) \le 9$$

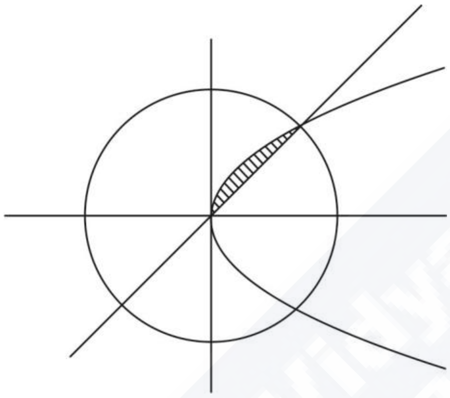
Using LMVT for $x \times [-7, 0]$

$$\frac{f(0)-f(-7)}{(0+7)} \le 2$$

$$\frac{f(0)+3}{7} \le 2 \quad \Rightarrow \quad f(0) \le 11$$

$\therefore \qquad f(0)+f(-1) \le 20$

15.(2)



Required area $=$ Total area $-$ shaded area

$$= 2\pi - \int_0^1 (\sqrt{x}-x)dx = 2\pi - \left(\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right)_0^1 = 2\pi - \left(\frac{2}{3}-\frac{1}{2}\right) = \frac{1}{6}(12\pi - 1)$$

16.(4) Distance between foci $= 2ae = 6$

$$ae = 3 \qquad \text{........ (i)}$$

Distance between directrixes $= \frac{a}{e} - \left(-\frac{a}{2}\right)$

$$\Rightarrow \quad \frac{2a}{e} = 12 \quad \Rightarrow \quad \frac{a}{e} = 6 \qquad \text{........ (ii)}$$

Multiply (i) and (ii)

$$a^2 = 18$$

Dividing (i) by (ii)

$$e^2 = \frac{1}{2}$$

Now, $\quad e^2 = 1 - \frac{b^2}{a^2}$

$$\frac{1}{2} = 1 - \frac{b^2}{18} \quad \Rightarrow \quad b^2 = 9$$

Length of $LR = \frac{2b^2}{a} = \frac{2\times 9}{3\sqrt{2}} = 3\sqrt{2}$

17.(4) Let the numbers $a-2d, a-d, a, a+d, a+2d$

Sum $=5a=25 \quad \Rightarrow \quad a=5$

Product $=(a^2-4d^2)(a^2-d^2)a=2520$

$$(25-4d^2)(25-d^2)5=2520$$

$$(4d^2-25)(d^2-25)=504$$

Solving we get $d=1$ or $d=\frac{11}{2}$

Now $d=\frac{11}{2}$, since $a-d=5-\frac{11}{2}=-\frac{1}{2}$

Therefore, largest number $=a+2d=5+2\left(\frac{11}{2}\right)=16$

18.(1) $g(x)=x^2+x-1$

$$g(f(x))=f^2(x)+f(x)-1$$

$$g\left(f\left(\frac{4}{5}\right)\right)=f^2\left(\frac{5}{4}\right)+f\left(\frac{5}{4}\right)-1$$

Also, $\quad g(f(x))=4x^2-10x+5$

$$g\left(f\left(\frac{5}{4}\right)\right)=4\times\left(\frac{5}{4}\right)^2-10\times\frac{5}{4}+5=\frac{25}{4}-\frac{50}{4}+5=-\frac{5}{4}$$

$\therefore \qquad f^2=\left(\frac{5}{4}\right)-1=-\frac{5}{4}$

$$\left(f\left(\frac{5}{4}\right)+\frac{1}{2}\right)^2-\frac{1}{4}-1=-\frac{5}{4}$$

$$\left(f\left(\frac{5}{4}\right)+\frac{1}{2}\right)^2=0 \quad \Rightarrow \quad f\left(\frac{5}{4}\right)=-\frac{1}{2}$$

19.(1 & 3) $f(a+b+1-x)=f(x) \quad \forall x\in R$

$$I=\frac{1}{a+b}\int_a^b x(f(x)+f(x+1))dx \qquad \textbf{. . . (i)}$$

$$I=\frac{1}{a+b}\int_a^b (a+b-x)\,(f(a+b-x)+f(a+b+1-x))dx \qquad \textbf{. . . (ii)}$$

$$I=\frac{1}{a+b}\int_a^b (a+b-x)\,(f(x+1)+f(x))\,dx \qquad \textbf{. . . (iii)}$$

(i) + (iii)

$$2I=\frac{a+b}{a+b}\left[\int_a^b (f(x+1)+f(x)\right)dx$$

$$2I=\int_a^b f(x+1)\,dx+\int_a^b f(x)dx$$

$$2I = \int_a^b f(a+b+1-x) + \int_a^b f(x)\,dx$$

$$I = \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$\int_a^b f(x)dx = \int_a^b f(x+1)dx$$

$$I = \int_{a-1}^{b-1} f(x+1)\,dt = \int_{a+1}^{b+1} f(x)\,dx$$

20.(2) Since $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$,

$\therefore \quad \vec{a} = \lambda(\hat{b} \pm \hat{c}) = \lambda\left(\dfrac{\hat{i}+\hat{j}}{\sqrt{2}} \pm \dfrac{\hat{i}-\hat{j}+4\hat{k}}{3\sqrt{2}}\right)$

$\therefore \quad \vec{a} = \lambda\left(\dfrac{4\hat{i}+2\hat{j}+4\hat{k}}{3\sqrt{2}}\right)$ or $\vec{a} = \lambda\left(\dfrac{2\hat{i}+4\hat{j}-4\hat{k}}{3\sqrt{2}}\right)$

On comparing, we get

$\alpha = 4, \beta = 4$ or $\alpha = 1, \beta = -2$

$\therefore \quad \vec{a} = 4\hat{i}+2\hat{j}+4\hat{k}$ or $\vec{a} = \hat{i}+2\hat{j}-2\hat{k}$

$\therefore \quad \vec{a}\cdot\vec{k}+2 = -2+2 = 0$

SECTION – 2

21.(36) $\lim\limits_{x\to 2} \dfrac{3^x + 3^{3-x} - 12}{3^{-x/2} - 3^{1-x}}$

$$\lim_{x\to 2} \frac{3^x + \frac{27}{3^x} - 12}{\frac{1}{3^{x-2}} - \frac{3}{3^x}}$$

$$\lim_{x\to 2} \frac{3^{2x} - 12\cdot 3^x + 27}{3^{x/2} - 3}$$

$$\lim_{x\to 2} \frac{(3^x-3)(3^x-9)}{(3^{x/2}-3)}$$

$$\lim_{x\to 2} \frac{(3^x-3)((3^{x/2})^2-(3)^2)}{3^{x/2}-3}$$

$$\lim_{x\to 2} \frac{(3^x-3)(3^{x/2}-3)(3^{x/2}+3)}{(3^{x/2}-3)} = (3^2-3)(3+3) = 36$$

22.(30) Let $(1-x+x^2......)(1+x+x^2+.......)=a_0+a_1x+a_2x^2+.......$

Put $x=1$

$$1(2n+1)=a_0+a_1+a_2+......+a_{2n}$$

Put $x=-1$

$$(2n+1)(1)=a_0-a_1+a_2-a_3.......$$

$$4n+2=2(a_0+a_2+......+a_{2n})$$

$$a_0+a_2+......+a_{2n}=2n+1$$

$$2n+1=61 \quad \Rightarrow \quad n=30$$

23.(18) $Var(1, 2,......n)=10$

$$\left(\frac{1^2+2^2+3^2+......+n^2}{n}\right)-\left(\frac{1+2+......+n}{n}\right)^2=10$$

$$\frac{n(n+1)(2n+1)}{6n}-\left(\frac{n(n+1)}{2n}\right)^2=10$$

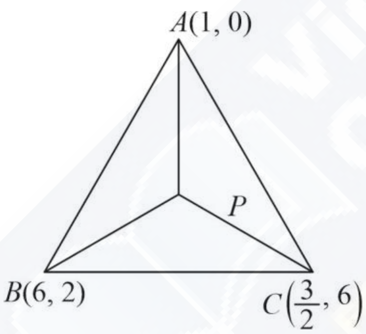
$$n^2-1=120 \quad \Rightarrow \quad n=11$$

Now, $Var(2, 4, 6......2m)=16$

$\therefore$ $Var(1, 2, 3......m)=4$

$\therefore$ $m+n=18 \quad \Rightarrow \quad m^2-1=48 \quad \Rightarrow \quad m=7$

24.(5)



Since $\Delta APC, \Delta APB$ and ΔBPC have equal area, P is a centroid of ΔABC.

$\therefore$ $P\equiv\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)=\left(\frac{17}{6}, \frac{8}{3}\right)$

$\therefore$ Length of line segment $PQ=\sqrt{\left\{\frac{17}{6}-\left(-\frac{7}{6}\right)\right\}\left(\frac{8}{3}-\left(-\frac{1}{3}\right)\right)^2}=5$units

25.(3) $f(x)=|2-|x-3||$

Clearly, $f(x)$ is non differentiable at points $x=1, x=3$ and $x=5$

$$f(f(x))=|2-||2-|-3||-3||$$

$\therefore$ $\sum_{x\in S} f(f(x))=f(f(-1))+f(f(3))+f(f(5))=1+1+1=3$