Vidyamandir Classes
Gurukul for IITJEE & Medical Preparation

SOLUTIONS

JEE Main – 2020 | 8th January 2020 (Evening Shift)

PHYSICS

SECTION – 1

1.(4) $R^3(2-R)=1$ i.e. $R^4-2R^3+1=0$

$R=1$ can't be solution so,

Remaining part is $\left(R^3-R^2-R-1\right)(R-1)=0$

i.e. R must satisfy $R^3-R^2-R-1=0$

2.(2) $\lambda_0=\dfrac{h}{mv_0\sqrt{2}}$

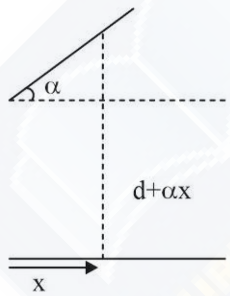
$\vec{V}=V_0\hat{i}+V_0\hat{j}-\dfrac{eE_0t}{m}\hat{k}$

$\lambda=\dfrac{h}{m|\vec{v}|}=\dfrac{\lambda_0}{\sqrt{1+\dfrac{e^2E^2t^2}{2m^2v_0^2}}}$

3.(1) $\dfrac{10}{R+100}=1mA$.

$R=9.9K\Omega$

4.(1)



$c=\int_0^a\dfrac{\in_0 adx}{d+\alpha x}=\dfrac{\in_0 a}{\alpha}ln\left(1+\dfrac{a\alpha}{d}\right)\approx\dfrac{\in_0 a}{\alpha}\left(\dfrac{a\alpha}{d}-\dfrac{a^2\alpha^2}{2d^2}\right)\approx\dfrac{\in_0 a^2}{d}\left(1-\dfrac{a\alpha}{2d}\right)$

5.(2) It is uniform circular motion.

6.(4) $I_p=I+I+2I\cos\phi$

$\phi=\dfrac{2\pi}{\lambda}.\dfrac{\lambda}{8}=\dfrac{\pi}{4}$

$I_p=2I\left(1+\dfrac{1}{\sqrt{2}}\right)$

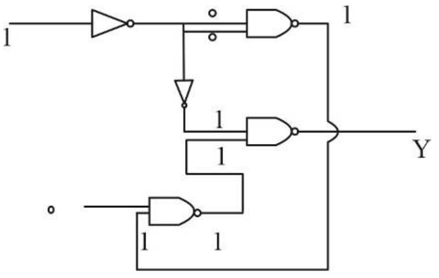
$I_m=4I$

$Ratio=\dfrac{1+\dfrac{1}{\sqrt{2}}}{2}=0.85$

7.(3) $\sqrt{T} \propto V$

$$T_2 = \frac{2.06 \times 10^5}{4} \equiv 5.15 \times 10^3 N$$

8.(3)



9.(4) $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow g = \frac{4\pi^2 \ell}{T^2}$

$$\frac{\Delta g}{g} = \frac{\Delta \ell}{t} + 2\frac{\Delta T}{T} = \frac{0.1}{25} + 2\left(\frac{1}{50}\right)$$

$$\frac{\Delta g}{g} \times 100 = 0.4 + 4 = 4.4\%$$

10.(4) $K.E. = \frac{7}{10} mv^2 = 8.75 \times 10^{-4} J$

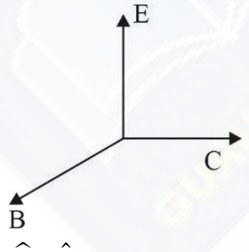
So, option is (4)

11.(3) $\frac{F_v}{F_D} = \frac{\rho_1 g.2.5 \times 50}{(\rho_1 g 5 + \rho_2 g \times 2.5) \times 50} = \frac{1}{4}$

12.(4) $i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)$

$$Q = \frac{E}{R}\int_0^{\tau}\left(1 - e^{-t/\tau}\right)dt = \frac{E}{R}\left(\tau - \tau\left[e^{-\tau/\tau}\right]_{\tau}^{0}\right) = \frac{E}{R}\left(\tau - \tau\left(1 - \frac{1}{e}\right)\right) = \frac{E}{R}\frac{\tau}{e} = \frac{EL}{eR^2}$$

13.(1)



$\widehat{E} = \hat{i}$

$E = BC = 15$

14. (4) $\frac{kQ_1 / R_1^2}{kQ_2 / R_2^2} = \frac{R_1}{R_2} \Rightarrow \frac{Q_1}{Q_2} = \left(\frac{R_1}{R_2}\right)^3$

$$\frac{kQ_1 / R_1}{kQ_2 / R_2} = \left(\frac{R_1}{R_2}\right)^2$$

15.(2) $V^2 = 2ax$

16.(2) Total time to land (using C.O.M)

$$-\frac{h}{2}=\frac{\sqrt{2gh}}{2}.T-\frac{1}{2}gT^2 \Rightarrow T=\left(\frac{\sqrt{2}+\sqrt{6}}{2}\right)\sqrt{\frac{h}{g}}$$

Time to collide $t=\frac{h}{\sqrt{2gh}}=\frac{1}{\sqrt{2}}\sqrt{\frac{h}{g}}$

So, time for combined mass to land $=T-t=\frac{\sqrt{2}+\sqrt{6}}{2}-\frac{1}{\sqrt{2}}=\frac{2+\sqrt{12}-2}{2\sqrt{2}}=\frac{\sqrt{6}}{\sqrt{4}}=\sqrt{\frac{3}{2}}$

17.(4) $C_v(mix)=\frac{n.\frac{3R}{2}+2n\frac{5R}{2}}{3n}=\frac{13R}{6}$

$C_p(mix)=\frac{19R}{6}$; $Y=\frac{19}{13}$

18.(4) $m=\frac{f}{x-f}$ f: mag of focal length

m = mag of magnification.

19.(2) $\frac{Q_H-Q_L}{Q_H}=\frac{1}{10}$

$\frac{W}{Q_H}=\frac{1}{10}\Rightarrow Q_H=100$

$Q_H-Q_L=10$

$100-Q_L=10\Rightarrow Q_L=90$

20.(4) $B_{(out)}=\frac{\mu_0 I}{4\pi R}\left(\frac{2}{\sqrt{2}}\right)+\frac{\mu_0 I}{2R}=\frac{\mu_0 I}{2\pi R}\left(\pi+\frac{1}{\sqrt{2}}\right)$

SECTION – 2

21.(8.00) $100=\frac{1}{2}gt^2$

$81=\frac{1}{2}g(t-\frac{1}{2})^2 \Rightarrow \frac{10}{9}=\frac{t}{t-0.5} \Rightarrow t=5 \quad \therefore \quad g=\frac{200}{t^2}=\frac{200}{25}=8$

22.(486) For Balmer series,

Transition happen to $n=2$ from higher orbits.

For transition $n=3 to n=2$

$E_3-E_2=\frac{hc}{\lambda_1} \Rightarrow -13.6\left[\frac{1}{3^2}-\frac{1}{2^2}\right]=\frac{hc}{(6561\text{Å})}$ ……..(i)

For transition $n=4$ to $n=2$

$E_4-E_2=\frac{hc}{\lambda_2} \Rightarrow -13.6\left[\frac{1}{4^2}-\frac{1}{2^2}\right]=\frac{hc}{\lambda_2}$ …….(ii)

$(i)/(ii)$

$$\frac{\lambda_2}{6561}=\frac{\left(\frac{1}{9}-\frac{1}{4}\right)}{\left(\frac{1}{16}-\frac{1}{4}\right)}=\frac{20}{27}$$

$$\lambda_2=\frac{20}{27}\times 6561=4860\text{Å}=486nm$$

23.(50) Let containers have temperature T_1, T_2 and $T_3 °C$ respectively.

For case: 1

$1 \times \rho_w \times c_w (T - T_1) + 2\rho_w c_w (T - T_2) = 0$

$3T - T_1 - 2T_2 = 0$

$T_1 + 2T_2 = 180.......(i) \quad [\ common\ temperature\ T = 60°]$

For case II

$1 \times \rho_w \times c_w (T - T_2) + 2 \times \rho_w c_w (T - T_3) = 0$

$3T - T_2 - 2T_3 = 0$

$T_2 + 2T_3 = 90 \quad [common \quad temperature\ T = 30°\ in\ this\ csse]$ …….(ii)

For case III:

$2\rho_w c_w (T - T_1) + 1\rho c_w (T - T_3) = 0$

$3T - 2T_1 - T_3 = 0$

$\Rightarrow 2T_1 + T_3 = 180°.........(iii) \quad [common\ temperature\ \ T = 60°\ in\ this\ case]$

For Case: -IV

$1\rho_w c_w (\theta - T_1) + 1\rho_w c_w (\theta - T_2) + 1\rho_w c_w (\theta - T_3) = 0$

$3\theta - T_1 - T_2 - T_3 = 0 \quad ; \quad \theta = \dfrac{T_1 + T_2 + T_3}{3}$

$(i) + (ii) + (iii) \Rightarrow 3(T_1 + T_2 + T_3) = 450°C \qquad \therefore\ \theta = \dfrac{T_1 + T_2 + T_3}{3} = \dfrac{450}{9} = 50°C$

24.(16) From energy conservation $U_i + K_i = U_f + K_f$

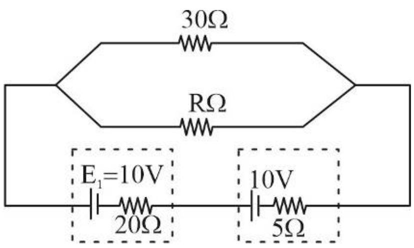
$$-\frac{GM_e m}{10R} + \frac{1}{2} m(v_i^2) = -\frac{GM_e m}{R} + \frac{1}{2} m v_f^2$$

$$v_f^2 = \left(\frac{2GM_e}{R}\right)\left[1 - \frac{1}{10}\right] + v_i^2 = v_e^2\left(1 - \frac{1}{10}\right) + v_i^2 = (11.2)^2 \times \frac{9}{10} + (12)^2$$

$$v_f = \sqrt{256.896} = 16.02\, km/s$$

25.(30) Current through the batteries;

$$i = \frac{10 + 10}{20 + 5 + \left(\dfrac{30R}{30 + R}\right)} = \frac{20}{25 + \left(\dfrac{30R}{30 + R}\right)}$$



P.d. across 20Ω internal resistance battery;

$$= 10 - i \times 20 = 0 \Rightarrow i = \frac{1}{2} \quad \text{Or} \quad \frac{20}{25 + \dfrac{30R}{30 + R}} = \frac{1}{2}$$

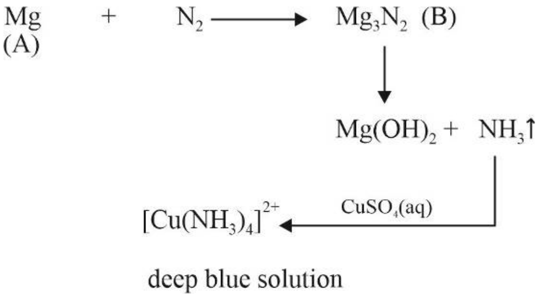
$$40 = 25 + \frac{30R}{30 + R}$$

$$(30 + R) \times 15 = 30R \quad ; \quad R = 30\Omega$$

CHEMISTRY

SECTION – 1

1.(4)



2.(2) AgBr shows both Frenkel and Schottky defects.

3.(1)
$FeO + SiO_2 \longrightarrow FeSiO_3$
$FeO \xrightarrow{\Delta} Fe + \frac{1}{2}O_2$
These reactions don't occur in the blast furnace during extraction of Fe

4.(4) Kjeldahl's method can't be used for Nitro compounds.

5.(4) $K = Ae^{-Ea/RT}$

$\log k = \log A - \frac{E_a}{2.303RT}$; $\text{slope} = \frac{-E_a}{2.303R}$

$\therefore$ In the given graph, greater the slope, more is the E_a $\therefore$ Order $= E_c > E_a > E_d > E_b$

6.(1) Three isotopes of hydrogen are:

Protinum	Deuterium	Tritium
$\left({}^1_1H\right)$	$\left({}^2_1H\right)$	$\left({}^3_1H\right)$

No. of neutrons in ${}^1_1H = 0$

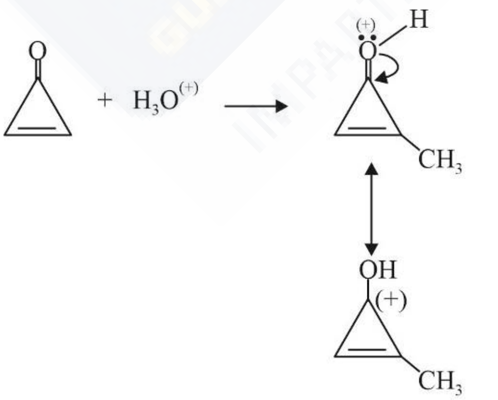
No. of neutrons in ${}^2_1H = 1$

No. of neutrons in ${}^3_1H = 2$

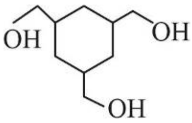
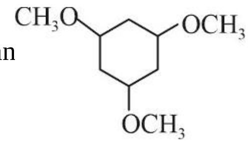
Total no. of neutrons $= 3$

7.(4) Catalytic hydrogenation depends upon the extent of adsorption.
Group 7-9 elements exhibit maximum adsorption property
The reactants must get adsorbed reasonably strongly on to the catalyst to become active. However, they must not get adsorbed so strongly that they are immobilized and other reactants are left with no space on the catalyst's surface for adsorption.
Hence, assertion is true but reason is false.

8.(2)



(Aromatic highly stable)

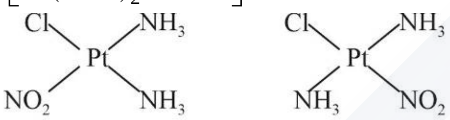
9.(1)  has greater boiling point than  than because of extensive –H bonding.

10.(3) Greater the bond length, lesser is the bond energy.
Order of bond length: $C-I > C-Br > C-Cl > C-F$
Order of bond energy: $C-F > C-Cl > C-Br > C-I$

11.(1) More the no. of shells, greater is the atomic radius.
$\therefore$ Size of Br $>$ Size of Cl
While moving along the period, the atomic size decreases due to increase in effective nuclear charge.
$\therefore$ Size: $C > O > F$
Overall order: $Br > Cl > C > O > F$

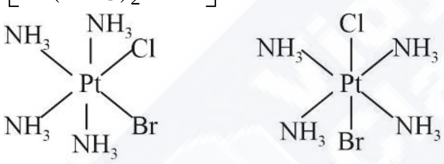
12.(4)

Compound	Geometry	Geometrical isomerism
$\left[Pt(NH_3)_3Cl\right]^+$	Square planar	×
$\left[Pt(NH_3)Cl_5\right]^-$	Octahedral	×
$\left[Pt(NH_3)_2ClNO_2\right]$	Square Planar	✓



'cis' 'trans'

$\left[Pt(NH_3)_2ClBr\right]^{2+}$	Octahedral	✓



'cis' 'trans'

13.(3) Bohr's radius $= a_0\dfrac{n^2}{Z}$

For Li^{2+}, $n = 2$ and $z = 3$ $\therefore v = \dfrac{4a_0}{3}$

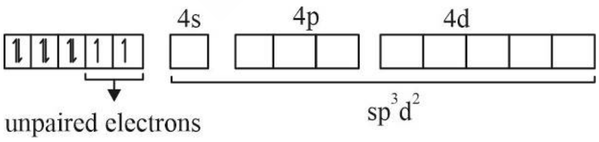
14(4) In $Ni(CO)_4$, CO is SFL $\therefore$ pairing occurs ; $\vec{\mu} = 0$

Similarly for $\left[Ni(CN)_4\right]^{2-}$ and $PdCl_2(PPh_3)_2$, under the effect of SFL, all e^-s are paired, hence $\vec{\mu} = 0$

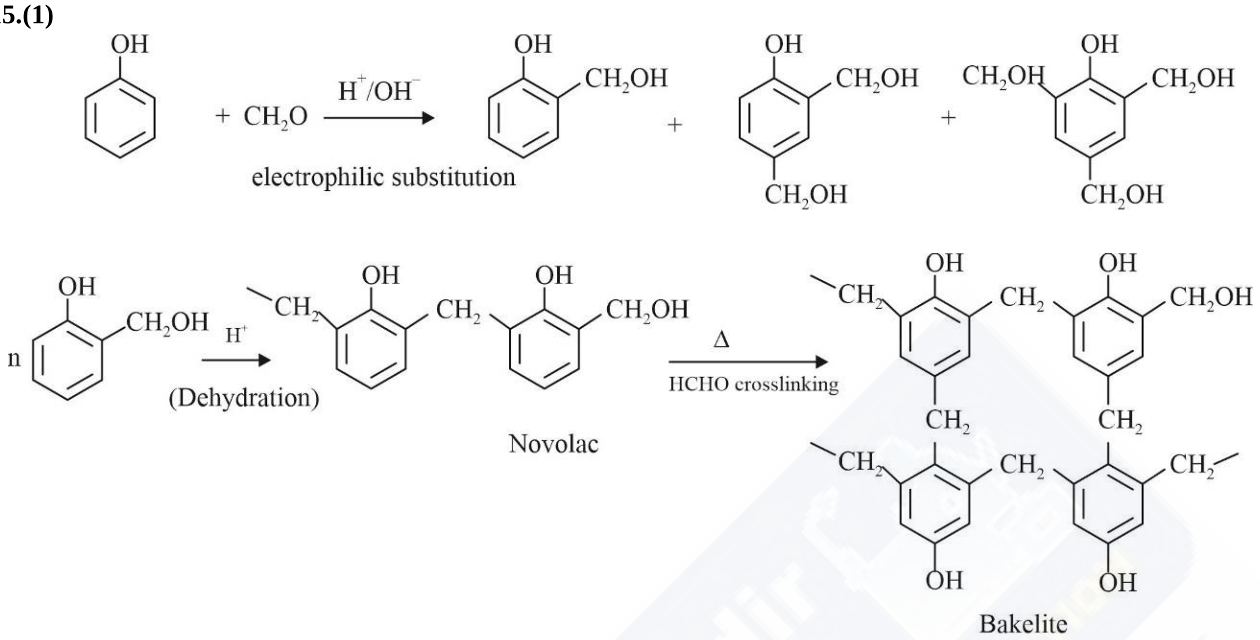
In $\left[Ni(H_2O)_6\right]Cl_2$ or $\left[Ni(H_2O)_6\right]^{2+}$,

H_2O is a weak field ligand,

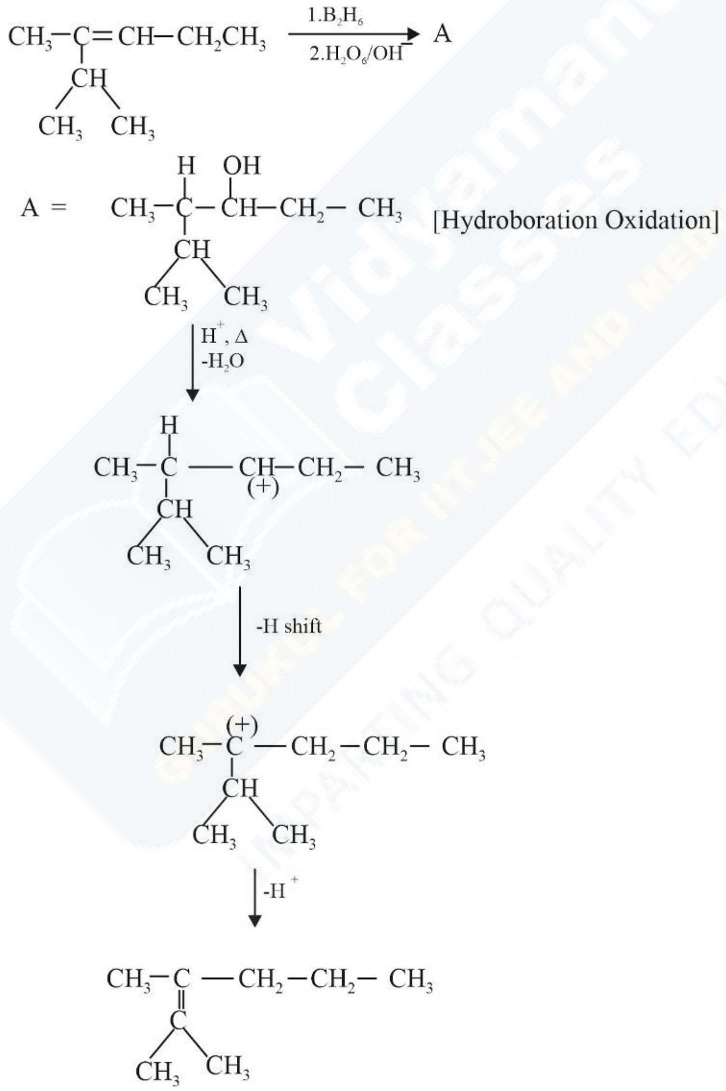
Ni^{2+}: d^8 configuration



15.(1)



16.(4)

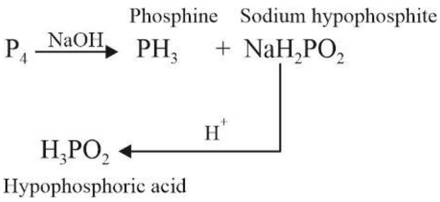


17.(3) $H_2O \rightleftharpoons H^+ + OH^-$

Dissociation of H_2O is an endothermic reaction.

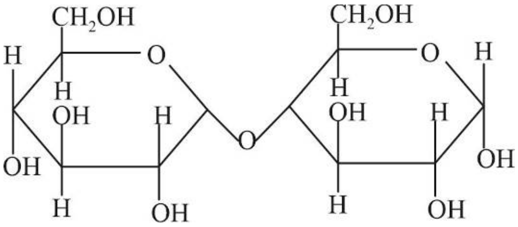
On increasing temperature, $\left[H^+\right]$ ion concentration increases. $\therefore$ pH decreases.

18.(3)



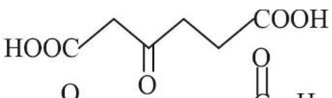
Basicity of $H_3PO_2 = 1$ (one replacable H^+ ion per molecule)

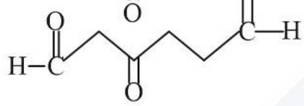
19.(2)

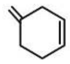


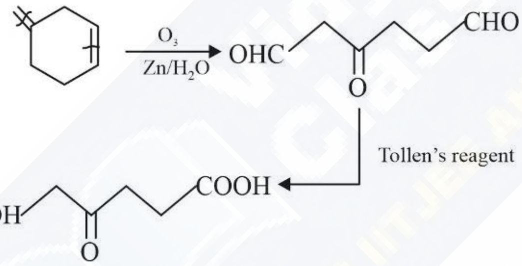
Maltose is a disaccharide of two $\alpha - D$ glucose monomers.

20.(2) $X \xrightarrow[ZN/H_2O]{O_3} A \xrightarrow{\left[Ag(NH_3)_2\right]^+} B$

B = 

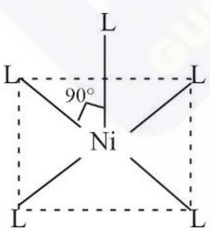
A = 

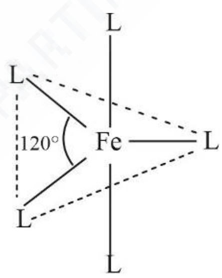
X = 



SECTION – 2

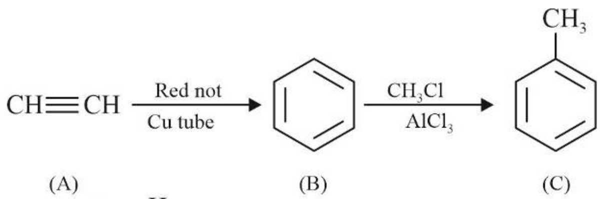
21.(20.00)

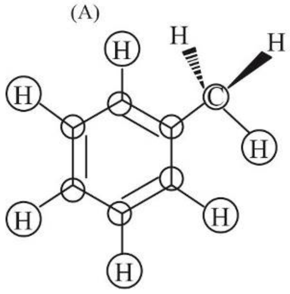




No. of 90° angles $= 8$	No. of 90°angles $= 6$
No. of 120° angles $= 0$	No. of 120°angles $= 3$
No. of 180° angles $= 2$	No. of 180°angles $= 1$
Total $= 20$	

22(13.00) $A = CH \equiv CH$





Max no of atoms in one plane $= 13$

23.(2130.00) $NaClO_{3(s)} + Fe_{(s)} \longrightarrow O_{2(g)} + NaCl_{(s)} + FeO_{(s)}$

No. of moles of $O_2 =$ no. of moles of $NaClO_3$

No. of moles of $O_2(n) = \frac{PV}{RT} = \frac{1 \times 492}{0.082 \times 300} = 20$

$\therefore$ Molar mass of $NaClO_3 = 23 + 35.5 + 48 = 106.5$

$\therefore$ Mass of $NaClO_3$ required $= 20 \times 106.5 = 2130g$

Ans. 2130

24.(2.15) $Sn(s) | Sn^{2+}(aq, 1M) || Pb^{2+}(aq, 1M) | Pb(s)$

Overall cell reaction $Sn_{(s)} + Pb^{2+}{}_{(aq)} \longrightarrow Sn^{2+}{}_{(aq)} + Pb_{(s)}$

$$E = E^\circ - \frac{0.06}{2} \log \frac{[Sn^{2+}]}{[Pb^{2+}]}$$

$E^\circ = -0.13 + 0.14 = 0.01V$

At equilibrium, $E = 0$

$$0 = 0.01 - 0.03 \log \frac{[Sn^{2+}]}{[Pb^{2+}]}$$

$$0.01 = 0.03 \log \frac{[Sn^{2+}]}{[Pb^{2+}]}$$

$$\frac{1}{3} = \log \frac{[Sn^{2+}]}{[Pb^{2+}]}$$

$$\frac{[Sn^{2+}]}{[Pb^{2+}]} = 10^{1/3} = 2.15$$

25.(6.25) $\Delta U = nC_v \Delta T$

$500 = 4 \times C_v \times (500 - 300)$

$C_v = \frac{5000}{4 \times 200} = 6.25 JK^{-1} mol^{-1}$

MATHEMATICS

SECTION – 1

1.(4) $f(x)=\begin{cases}\dfrac{x}{1+x^2} & x\in(1,2)\\ \dfrac{2x}{1+x^2}, & x\in[2,3)\end{cases}$

$f'(x)\begin{cases}\dfrac{(1-x^2)}{(1+x^2)}<0 & x\in(1,2)\\ \dfrac{2(1-x^2)}{1+x^2}<0 & x\in[2,3)\end{cases}$

So, the function is decreasing in (1, 2) and [2, 3) therefore, range is $\left(\frac{2}{5},\frac{1}{2}\right)\cup\left(\frac{3}{4},\frac{4}{5}\right]$

2.(4) As given $\bar{x}=\dfrac{\sum x_i}{20}=10\Rightarrow\sum x_i=200$

Similarly, $\sigma^2=\dfrac{\sum x_i^2}{20}-(\bar{x})^2=\dfrac{\sum x_1^2}{20}-100=4\Rightarrow\sum x_1^2=2080$

Final mean after correction $=\dfrac{200-9+11}{20}=\dfrac{202}{20}$

Final variance after correction $=\dfrac{2080-(9)^2+(11)^2}{20}-\left(\dfrac{202}{20}\right)^2=\dfrac{2120}{20}-(10.1)^2=3.99$

3.(2) $\lim\limits_{x\to0}\dfrac{\int_0^x t\,sin(10t)\,dt}{x}$; $\lim\limits_{x\to0}\dfrac{x\,sin(10x)}{1}=0$

4.(3) $\vec{b}\times\vec{c}=\vec{b}\times\vec{a}\Rightarrow\vec{b}\times(\vec{c}-\vec{a})=0$

$\vec{c}-\vec{a}=\lambda\vec{b}$; $\vec{a}.\vec{c}-\vec{a}.\vec{a}=\lambda\vec{b}.\vec{a}$; $0-6=\lambda(4)$; $\lambda=-\frac{3}{2}$

$\vec{c}-\vec{a}=-\frac{3}{2}(\hat{i}-\hat{j}+\hat{k})$; $\vec{c}.\vec{b}-\vec{a}.\vec{b}=-\frac{3}{2}(\hat{i}-\hat{j}+\hat{k})(\hat{i}-\hat{j}+\hat{k})$; $\vec{c}.\vec{b}-4=-\frac{3}{2}\times3$

$\vec{c}.\vec{b}-4=-\frac{9}{2}$; $\vec{c}.\vec{b}=4-\frac{9}{2}=-\frac{1}{2}$

5.(3) Given curve is $x^2+2xy-3y^2=0$

$x^2+3xy-xy-3y^2=0$

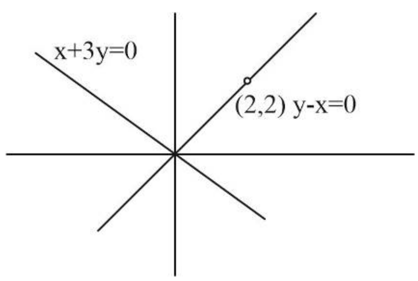
$x(x+3y)-y(x+3y)=0$

$(x-y)(x+3y)=0$

Equation of line $\perp$ to $y-x=0$ and passing through $(2,2)$

$y-2=-1(x-2)\Rightarrow x+y=4$

Its distance from origin $\left|\dfrac{0+0-4}{\sqrt{2}}\right|=2\sqrt{2}$



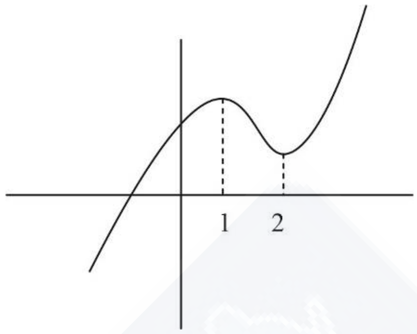
6.(2) Given $I = \int_1^2 \frac{dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$

Say $f(x) = 2x^3 - 9x^2 + 12x + 4$

$f'(x) = 6x^2 - 18x \ \ +12$

$f(1) = 9 \ and \ f(2) = 8$

$f(x)$ decreases from $x = 1 to\, x = 2$



So $\frac{1}{\sqrt{2x^3 - 9x^2 + 12x + 4}}$

Increases from $x = 1, to\, x = 2$

$\frac{1}{3} < \frac{1}{\sqrt{2x^3 - 9x^2 + 12x + 4}} < \frac{1}{2\sqrt{2}}$

$\int_1^2 \frac{1}{3} dx < \int \frac{dx}{\sqrt{2x^3 - 9x^2 + 12x + 4}} < \int \frac{dx}{2\sqrt{2}}$ So $I^2 \in \left(\frac{1}{9}, \frac{1}{8}\right)$

7.(4) $A = \begin{bmatrix} 2 & 2 \\ 9 & 4 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$|A - \lambda I| = 0$ characteristic equation

$\begin{vmatrix} 2-\lambda & 2 \\ 9 & 4-\lambda \end{vmatrix} = 0$

$(2-\lambda)(4-\lambda) - 18 = 0$

$\lambda^2 - 6\lambda + 8 - 18 = 0$

$\lambda^2 - 6\lambda - 10 = 0$

$A^2 - 6A - 10I = 0$

$A - 6I - 10A^{-1} = 0$

$10A^{-1} = A - 6I$

8.(1) $\alpha = w$; clearly $\alpha^3 = 1$ and $1 + \alpha + \alpha^2 = 0$

$a = (1+w)\sum_{k=0}^{100} w^{2k}, b = \sum_{k=0}^{100} \alpha^{3k} \Rightarrow b = 101$

$a = (1+w)\left(\frac{1 - w^{202}}{1 - w^2}\right)$

$a = \frac{1 - w^{202}}{1 - w} = \frac{1 - w}{1 - w} = 1$

Hence equation having roots 1 and 101

$x^2 - 102x + 101 = 0$

9.(Bonus) For constant function

Which is continuous and differentiable in [0, 1]

Option 2, 3, 4 are false

$\dfrac{f(c)-f(1)}{C-1}=f'(d)$ by LMVT $d\in(c,1)$

and we can't compare $f'(d)$ and $f'(c)$ None option is correct

10.(1) Given equation is $3^x\left(3^x-1\right)+2=\left|3^x-1\right|+\left|3^x-2\right|$

Let $3^x=t, t>0$

$t^2-t+2=|t-1|+|t-2|$

$y=t^2-t+2$

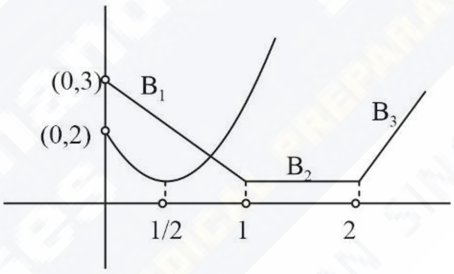
$y=|t-1|+|t-2|$

Point of intersections of the graphs of these two functions will be solution (for $t>0$)

B_1 will intersect at only one point

B_2 will not intersect

For B_3



$t^2-t+2=t-1+t-2$

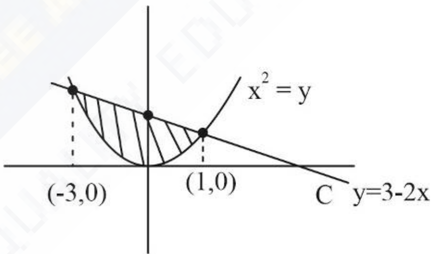
$t^2-t+2=2t-3$

$t^2-3t+5=0$

Roots are imaginary

So solution set is a singleton

11.(3) $\left\{(x,y)\in R^2 : x^2\le y\le 3-2x\right\}$



$x^2=3-2x$

$x^2+2x-3=0$

$x^2+3x-x-3=0$

$x=-3, x=1$

Required area $\displaystyle\int_{-3}^{1}\left((3-2x)-x^2\right)dx=\int_{-3}^{1}\left(3-2x-x^2\right)dx=\left[3x-x^2-\frac{x^3}{3}\right]_{-3}^{1}$

$=\left(3-1-\dfrac{1}{3}\right)-(-9-9+9)=\dfrac{32}{3}$

12.(1) Given system of equations

$\lambda x+2y+2z=5$

$2\lambda x+3y+5z=8$

$4x+\lambda y+6z=10$

$$\begin{vmatrix}\lambda & 2 & 2\\ 2\lambda & 3 & 5\\ 4 & \lambda & 6\end{vmatrix}=0$$

$\lambda(18-5\lambda)+2(20-12\lambda)+2\left(2\lambda^2-12\right)=0$

$\lambda^2+6\lambda-16=0$

$\lambda = 2\, and\, \lambda = -8$

Now, $D_1 = \begin{vmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{vmatrix}$

$D_1 \neq 0$

Thus for $\lambda = 2$, given set of equations has no solution.

13.(2) Given family of curves is

$x^2 = 4b(y+b), b \in R$.(i)

Differentiating both sides w.r.t x

$2x = 4b\frac{dy}{dx} \Rightarrow b = \frac{x}{2(dy)(dx)}$ ……..(ii)

On substituting the value of 'b' from (ii) to (i)

$$x^2 = 4\frac{x}{2\left(\frac{dy}{dx}\right)}\left(y + \frac{x}{2\left(\frac{dy}{dx}\right)}\right)$$

We get $x\left(\frac{dy}{dx}\right)^2 = 2y\left(\frac{dy}{dx}\right) + x$

14.(2) Mid point of point and its image is $\left(-\frac{2}{3}, \frac{1}{3}, \frac{4}{3}\right)$

Direction ratios of the normal $\left(\frac{10}{3}, \frac{10}{3}, \frac{10}{3}\right)$

So, the equation of plane is

$$\frac{10}{3}\left(x + \frac{2}{3}\right) + \frac{10}{3}\left(y - \frac{1}{3}\right) + \frac{10}{3}\left(z - \frac{4}{3}\right) = 0$$

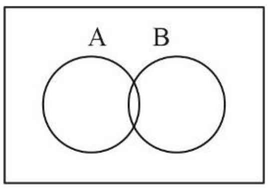
$x + y + z = 1$

$(1,-1,1)$ ratio for equation of the plane

15.(3) $P(A \cup B) = \frac{1}{2}$

$P(A) + P(B) - 2P(A \cap B) = \frac{2}{5}$ ……..(i)

$P(A) + P(B) - P(A \cap B) = \frac{1}{2}$ …….(ii)



(i) – (ii)

$-P(A \cap B) = \frac{2}{5} - \frac{1}{2}$

$-P(A \cap B) = -\frac{1}{10}$

$P(A \cap B) = \frac{1}{10}$

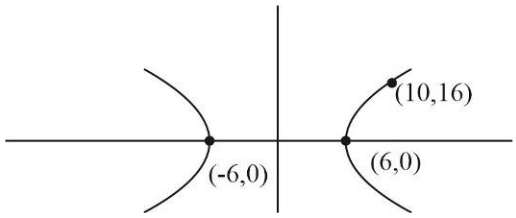
16.(1) $(\sim p \wedge q) \rightarrow (p \vee q)$

$\sim((\sim p \wedge q) \wedge (\sim p \wedge \sim q))$

17.(4) Let the equation of the hyperbola is $\frac{x^2}{36}-\frac{y^2}{b^2}=1$

Since it passes through $(10,16)$

$\frac{100}{36}-\frac{256}{b^2}=1 \Rightarrow b^2=144$



So the equation of the hyperbola is $\frac{x^2}{36}-\frac{y^2}{144}=1$

Equation of normal $\frac{x}{10/36}+\frac{y}{16/144}=36+144$

$2x+5y=100$

18.(2) $a+9d=\frac{1}{20}$ ....(i)

$a+19d=\frac{1}{10}$ ....(ii)

$10d=\frac{1}{10}-\frac{1}{20}$; $10d=\frac{1}{20}$; $d=\frac{1}{200}$

Now $a+\frac{9}{200}=\frac{1}{20}$; $a=\frac{1}{20}-\frac{9}{200}$; $a=\frac{1}{200}$

$S_{200}=\frac{200}{2}\left[\frac{2}{200}+199\times\frac{1}{200}\right]=100\times\frac{201}{200}=100\frac{1}{2}$

19.(3) $L_1 \equiv x+y=\sqrt{2}$

Since $y=mx+c$ is $\perp$ to $x+y=\sqrt{2}$

So $y=mx+c$ becomes $y=x+c \Rightarrow y-x-c=0$

Now $y-x-c=0$ is tangent to $(x-3)^2+y^2=1$

$\left|\frac{-3+0-c}{\sqrt{2}}\right|=1 \Rightarrow |3+c|=\sqrt{2}$

$c^2+6c+9=2$; $c^2+6c+7=0$

20.(4) $\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6=2\left\{{}^6C_0x^6+{}^6C_2x^4\left(x^2-1\right)+{}^6C_4x^2\left(x^2-1\right)^2+{}^6C_6\left(x^2-1\right)^3\right\}$

$=2\left\{x^6+15x^6-15x^4+15x^6-30x^4+15x^2+x^6-6x^4+3x^2-1\right\}$

$=2\left\{32x^6-48x^4+18x^2-1\right\}=16x^6-96x^4+36x^2-2$

$\alpha=-96; \beta=36; \alpha-\beta=-96-36=-132$

SECTION – 2

21.(2454) Given word is
EXAMINATION
2A, 2I, 2N, 1E, 1X, 1M, 1T, 1O

Case (i) two pairs $\rightarrow {}^3C_2\times\frac{4!}{2!2!}$

Case (ii) one pair two diff $\rightarrow {}^3C_1\times{}^7C_2\times\frac{4!}{2!}$

Case (iii) all diff $\rightarrow {}^8C_4\times 4!$

Total no of words $=18+756+1680=2454$

22.(3) Let $f(x)=ax^3+bx^2+cx+d,\ f'(x)=3ax^2+2bx+c$; $\quad f''(x)=6ax+2b$

$-a+b-c+d=10$ …(i)

$a+b+c+d=-6$ …….(ii)

$3a-2b+c=0$ ……(iii)

$6a+2b=0$ ….(iv)

So, $a=1, b=-3,\ c=-9, d=5$

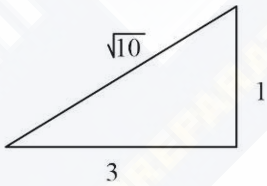
Also, for local minima $f'(x)=0 \quad \Rightarrow 3x^2-6x-9=0 \qquad \Rightarrow x=-1,3$

So local minima at $x=3$

23(1) $\dfrac{\sqrt{2}\,sin\alpha}{\sqrt{1+cos\,2\alpha}}=\dfrac{1}{7}\,and\sqrt{\dfrac{1-cos\,2\beta}{2}}=\dfrac{1}{\sqrt{10}} \quad \alpha,\beta\in\left(0,\dfrac{\pi}{2}\right)$

$$\frac{\sqrt{2}\,sin\alpha}{\sqrt{1+2cos^2\alpha-1}}=\frac{1}{7}$$

$$\frac{sin\alpha}{|cos\alpha|}=\frac{1}{7} \quad \alpha\in\left(0,\frac{\pi}{2}\right) \qquad ; \qquad tan\alpha=\frac{1}{7}$$



Now $\sqrt{\dfrac{1-1+2\,sin^2\beta}{2}}=\dfrac{1}{\sqrt{10}}$

$$|sin\beta|=\frac{1}{\sqrt{10}},\beta\in\left(0,\frac{\pi}{2}\right)$$

$$sin\beta=\frac{1}{\sqrt{10}},\ tan\beta=\frac{1}{3} \qquad ; \qquad tan2\beta=\frac{2/3}{1-\frac{1}{9}}=\frac{2/5}{\frac{8}{9}-3}=\frac{3}{4}$$

Now $tan(\alpha+2\beta)=\dfrac{tan\alpha+tan2\beta}{1-tan\alpha,tan2\beta}=\dfrac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\times\frac{3}{4}}=1$

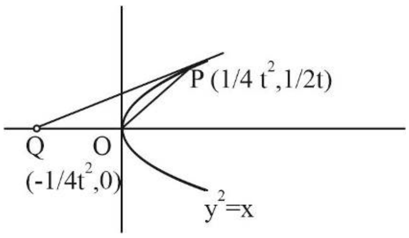
24.(504)

$$\sum_{n=1}^{7}\frac{n(n+1)(2n+1)}{4}$$

$$\frac{1}{4}\sum_{n=1}^{7}2n^3+3n^2+n \quad ; \quad \frac{1}{4}\left\{2\left(\frac{7\times 8}{2}\right)^2+3\left(\frac{7\times 8\times 15}{6}\right)+\frac{7\times 8}{2}\right\}$$

25.(0.5) Area of $\Delta OPQ=4$

Slope of OP $=\dfrac{2}{t}$



$$\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ -\frac{1}{4}t^2 & 0 & 1\\ \frac{1}{4}t^2 & \frac{1}{2}t & 1\end{vmatrix}=\pm 4$$

$$-\frac{1}{8}t^3=\pm 8$$

$$t^3=64\Rightarrow t=4 \qquad \text{So } m=\frac{1}{2}$$