Vidyamandir Classes
Gurukul for IITJEE & Medical Preparation

SOLUTIONS

JEE Main – 2020 | 8th January 2020 (Morning Shift)

PHYSICS

SECTION – 1

1.(3) $\tau = \dfrac{\text{Mean free path}}{V_{rms}}$

$$= \frac{\vec{v}\,t}{\underbrace{\sqrt{2}\pi d^2 \vec{v}\,t}_{\text{volume of inter action}} \underset{\text{Number of molecules per unit volume}}{nv}} \times \frac{\perp}{\sqrt{\dfrac{3RT}{M}}}$$

$$= \frac{\sqrt{M}}{\sqrt{3RT} \times \sqrt{2}\pi d^2 nv}; \qquad \tau \alpha \frac{\perp}{\sqrt{T}}$$

Correct answer (3)

2.(3) $C_1 + C_2 = 10\mu F$

$$\frac{1}{2}C_2 \times 1^2 = 4 \times \left(\frac{1}{2} \times C_1 \times 1^2\right)$$

$$C_2 = 4C_1$$

$$4C_1 + C_1 = 10\mu F$$

$$C_1 = 2\mu F$$

$$C_2 = 8\mu F$$

Equation $= \dfrac{C_1 C_2}{C_1 + C_2} = \dfrac{2 \times 8}{2 + 8} = 1.6\,\mu F$

Correct answer (3)

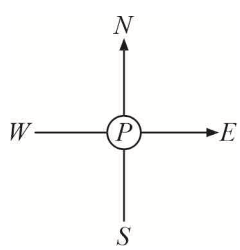
3.(3) $f_0 + f_e = 60$; $\qquad \dfrac{f_0}{f_e} = 5$

$\dfrac{60 - f_e}{f_e} = 5$; $\qquad f_e = 10\ cm$

4.(1) $\dfrac{1}{2} \times 1.6 \times 10^{-27} \times v^2 = 1 \times 10^6 \times 1.6 \times 10^{-19}$

$$v^2 = 2 \times 10^{14}; \qquad v = \sqrt{2} \times 10^7$$

$$10^{12} = \frac{1.6 \times 10^{-19} \times \sqrt{2} \times 10^7 \times B}{1.6 \times 10^{-27}}$$

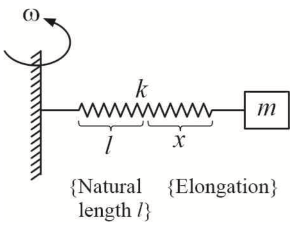


$$\frac{10^{-3}}{\sqrt{2}} = B\,]$$

$$B = 0.7 \times 10^{-3} T$$

Correct answer is (1)

5.(1)



$$m\omega^2(l+x)=kx$$

$$m\omega^2 l+m\omega^2 x=kx$$

$$\frac{m\omega^2 l}{k-m\omega^2}=x$$

Correct answer is (1)

6.(4) For x to y

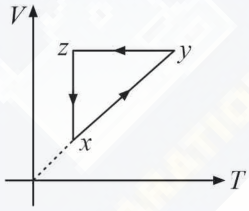
$V\alpha T$; $PV=\eta RT$

This means P is constant

Also volume is increasing

Only graph with pressure constant in process xy is (4).

Correct answer is (4)



7.(3) By conservation of angular momentum about hinge

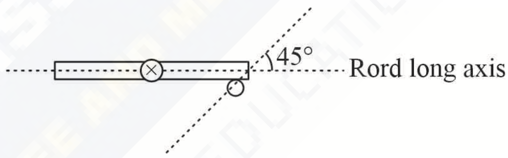
$$\frac{l}{2}\times mv\sin 45^\circ=\left\{\frac{4ml^2}{12}+m\left(\frac{l}{2}\right)^2\right\}\omega$$

$$\frac{mvl}{2}\times\frac{1}{\sqrt{2}}=\left\{\frac{ml^2}{3}+\frac{ml^2}{4}\right\}\omega$$

$$\frac{v}{2\sqrt{2}}=\left\{\frac{7l}{12}\right\}\omega$$

$$\omega=\frac{v\times 12}{2\sqrt{2}\times 7l}$$

$$\omega=\frac{3v\sqrt{2}}{7l}$$



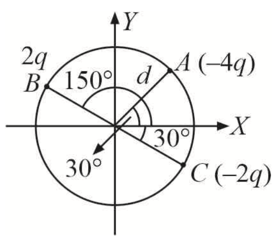
Correct answer is (3)

8.(2)

$$2\times\frac{k(2q)}{d^2}\cos 30\hat{i}+\frac{k(4q)}{d^2}\cos 30\hat{i}$$

$$\frac{4q\times 1}{4\pi\varepsilon_0 d^2}\times\frac{\sqrt{3}}{2}\hat{i}+\frac{4q\times 1}{4\pi\varepsilon_0 d^2}\times\frac{\sqrt{3}}{2}\hat{i}$$

$$\frac{\sqrt{3}q}{\pi\varepsilon_0 d^2}\hat{i}$$

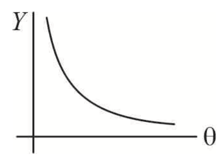


Correct answer is (2)

9.(1) In Ruther ford gold foil experiment, angle of scattering is negligible for most of α f particle, and very few scatter through large angle and extremely small number retrace its path.

$Y\alpha\frac{1}{\theta}$ (Note exactly but close to)

Correct answer is (1)



10.(3) $\lambda = \dfrac{h}{\sqrt{2(KE)m}} \Rightarrow \quad \lambda \propto \dfrac{1}{KE}$

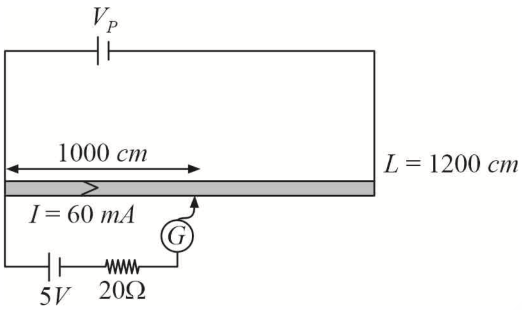
$$\frac{\lambda_A}{\lambda_B} = \sqrt{\frac{KE_B}{KE_A}} \quad \Rightarrow \quad \frac{1}{2} = \sqrt{\frac{T_A - 1.5}{T_A}} \quad \Rightarrow \quad T_A = 2eV$$

$$KE_B = 2 - 1.5 = 0.5eV$$

$$\phi_B = 4.5 - 0.5 = 4eV$$

Correct answer is (3)

11.(3)



Potential gradient $= \dfrac{5}{1000} = \dfrac{V_P}{1200}$

$V_P = 6V$

And $R_P = \dfrac{V_P}{I} = \dfrac{6}{60 \times 10^{-3}} = 100\ \Omega$

12.(3) Magnitude of electric field is constant and the surface is equipotential.

13.(1) $V = \dfrac{1}{\sqrt{\mu\varepsilon}}$

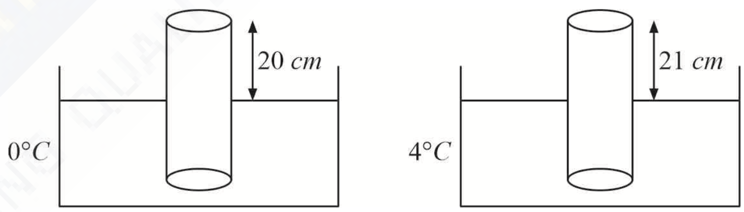
$n = \sqrt{\mu_r \varepsilon_r} = 2$

$\sin c = \dfrac{1}{2}; \qquad c = 30°$

14.(3) $mg = A(80)\rho_{0°C}\, g$

$mg = A(79)\rho_{4°C}\, g$

$\dfrac{\rho_{4°C}}{\rho_{0°C}} = \dfrac{80}{79} = 1.01$



15.(2) $\rho = \rho_0\left(1 - \dfrac{r^2}{R^2}\right) \qquad 0 < r \le R$

$mg = B$

$$\int \rho(4\pi r^2 dr) = \rho_L \frac{4}{3}\pi R^3$$

$$\int \rho_0\left(1 - \frac{r^2}{R^2}\right) 4\pi r^2 dr = \rho_L \frac{4}{3}\pi R^3$$

$$\int_0^R \rho_0 4\pi\left(r^2 - \frac{r^4}{R^2}\right) dr = \rho_0 4\pi\left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)_0^R = \rho_L \frac{4}{3}\pi R^3$$

$$\frac{2}{5}\rho_0 = \rho_L$$

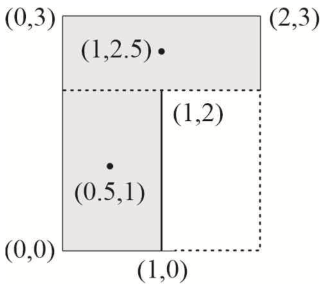
16.(2) First part of figure shown OR gate and
Second part of figure shown NOT gate
So Y_p = OR + NOT = NOR gate

$$Y = \overline{A+B} = \overline{A} \cdot \overline{B}$$

17.(3) $\varepsilon = \left| -\frac{d\varphi}{dt} \right| = \left| \frac{AdB}{dt} \right|$

$$= .(16 \times 4 - 4 \times 2)\frac{(1000-500)}{5} \times 10^{-4} \times 10^{-4} = 56 \times \frac{500}{5} \times 10^{-8} = 56 \times 10^{-6} V$$

18.(1)



$$\vec{r}_{cm} = \frac{1 \times \left(\frac{\hat{i}}{2} + \hat{j}\right) + 1 \times \left(\hat{i} + \frac{5\hat{j}}{2}\right)}{2}; \qquad \vec{r}_{cm} = \frac{3}{4}\hat{i} + \frac{7}{4}\hat{j}$$

19.(Bonus)

$V_0 = kh^x c^y G^z A^t$

$[ML^2T^{-3}A^{-1}] = [ML^2T^{-1}]^x [LT^{-1}]^y [M^{-1}L^3T^{-2}]^2 [A]^t$

$t = -1$ ….(1)

$x - 2 = 1$ …(2)

$2x + y + 32 = 2$ ….(3)

$-x - y - 22 = -3$ …(4)

(3) + (4) : $x + 2 = -1$ …(5)

(2) + (5) : $2x = 0 \Rightarrow x = 0$

$\therefore$ $2 = -1,\ y = 5\ \therefore$ No option is correct

20.(4) $3 = \frac{Gm_2}{2^2}$

$2 = \frac{Gm_1}{1^2}$ $\quad\therefore\quad$ $\frac{3}{2} = \frac{1}{4}\frac{m_2}{m_1};$ $\quad$ $\frac{m_1}{m_2} = \frac{1}{6}$

SECTION – 2

21.(580) For particle 1 $x = 10 + 8t - 3t^2$

$\therefore\ \overrightarrow{V_1} = (8 - 6t)\hat{i}$

For particle 2, $y = 5 - 8t^3$

$\overrightarrow{V_2} = (-24t^2)\hat{j}$

At $t = 1, \overrightarrow{V_2} = -24\hat{j},\ \overrightarrow{V_1} = 2\hat{i}$

$\therefore\ \overrightarrow{V_{21}} = \overrightarrow{V_2} - \overrightarrow{V_1} = -24\hat{j} - 2\hat{i}$

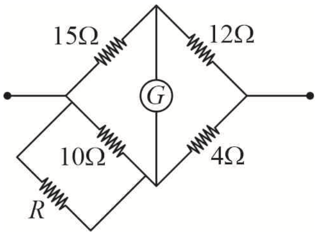
$|\overrightarrow{V_{21}}| = \sqrt{(24)^2 + (2)^2} = \sqrt{580}$

22.(1) $(0.1)(3\hat{i}) + (0.1)(5\hat{j}) = (0.1)4(\hat{i} + \hat{j}) + (0.1)\overrightarrow{V_B}$

$$\Rightarrow \overrightarrow{V_B} = 3\hat{j} + 5\hat{j} - 4\hat{i} - 4\hat{j} = -\hat{i} + \hat{j}$$

$$\therefore \ |\overrightarrow{V_B}| = \sqrt{2},\ K_B = \frac{1}{2}(0.1)(\sqrt{2})^2 = \frac{1}{10}$$

23.(10)



$$\frac{10R}{10+R} \times 12 = 15 \times 4$$

On solving

$$R = 10\ \Omega$$

24.(60) $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$R_1 = \infty$$

$$R_2 = -30\ cm$$

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{\infty} - \frac{1}{-30}\right)$$

$$\frac{1}{f} = \frac{0.5}{30}$$

$$f = 60\ cm$$

25.(106.05-106.07)

$$v = \sqrt{\frac{\beta}{\rho}}$$

$$\frac{v_{pipe}}{v_{air}} = \frac{\sqrt{\beta/2\rho}}{\sqrt{\beta/\rho}} = \frac{1}{\sqrt{2}}$$

$$v_{pipe} = \frac{v_{air}}{\sqrt{2}}$$

$$f_n = \frac{(n+1)v_{pipe}}{2l}$$

$$f_1 - f_0 = \frac{v_{pipe}}{2l} = \frac{300}{2\sqrt{2}}$$

→ 105.75 Hz (For $\sqrt{2} = 1.41$)

→ 106.05 Hz (For $\sqrt{2} = 1.414$)

CHEMISTRY

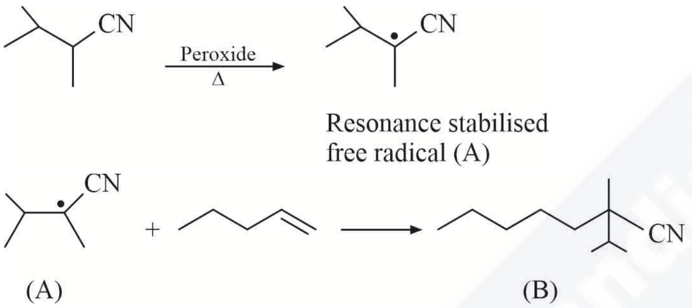
SECTION – 1

1.(1) $[Co(NH_3)_3(NO_2)_3] \rightarrow [Ma_3b_3]$ type

If three donor atoms of the same ligands occupy adjacent positions at the corners of an octahedral, we have the facial (fac) isomer. When the positions are around the meridian of the octahedron, we get the meridional (mer) isomer.

2.(2) Ethyl acetate ($CH_3-\overset{\overset{O}{\|}}{C}-O-C_2H_5$) is polar molecule, so dipole-dipole and London dispersion forces are present in it.

3.(3)



4.(4) When gypsum is heated to 393 K, it forms hemihydrate of calcium sulphate (plaster of paris).

5.(1) Diborane reduces carboxylic acid to 1°-alcohols.

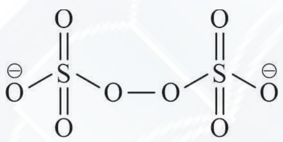
6.(1) E^1 reaction proceeds with Carbocation intermediate, therefore greater the stability of carbocation, faster is the rate of reaction.

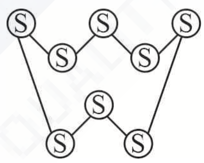
So rate of

7.(4) $Fe^{+2} \rightarrow [Ar]_{18} 4s^\circ 3d^6$

So, it will achieve $3d^5$ (stable configuration) after removal of 1 electron from Fe^{+2}.

8.(4)





($S_2O_8^{2-}$)
(Number of S — O bonds = 8)

(rhombic sulphur)
(Number of S — S bonds = 8)

9.(2) According to Hardy-Schulze rule,

Coagulation value or flocculation value $\propto \dfrac{1}{\text{Coagulation power}}$

Example

In the Coagulation of the positive sol, the flocculating power is in the order–

$[Fe(CN)_6]^{4-} > PO_4^{3-} > SO_4^{2-} > Cl^-$

10.(2) $CH_3OH \rightarrow$ No resonance

In p-Ethoxyphenol, due to +R effect of $-O-C_2H_5$ group, resonance will be less as compared to phenol.

So, C — OH bond length will be

Phenol < p-ethoxyphenol < methanol

11.(4) $\therefore \quad K = Ae^{-Ea/RT}$ …(1)

$\therefore \quad 10^6 \times K = Ae^{(-E_a)_{cat}/RT}$ …(2)

Dividing equation (2) by (1),

$10^6 = e^{E_a-(E_a)_{cat}/RT} \quad \Rightarrow \quad 6 \times \ln 10 = \dfrac{E_a-(E_a)_{cat}}{RT}$

$\Rightarrow \quad \Delta E_a = E_a - (E_a)_{cat} = 6 \times \ln 10 \times RT \quad \Rightarrow \quad (E_a)_{cat} - E_a = -6 \times 2.303 \times RT$

12.(4) $\bar{v} = R_H\left(\dfrac{1}{n_1^2}-\dfrac{1}{n_2^2}\right)$

$\Rightarrow$ As wavelength decrease, the lines in the series converge.

$\Rightarrow$ For Balmer series, $n_1 = 2$

$\Rightarrow$ For Balmer series, the lines of longest wavelength corresponds to $n_2 = 3$.

13.(2) $\rightarrow$ Glucose exists in two crystalline forms α and β

$\rightarrow$ Glucose does not give the schiff's test for aldehyde

$\rightarrow$ Glucose penta-acetate does not react with hydroxylamine

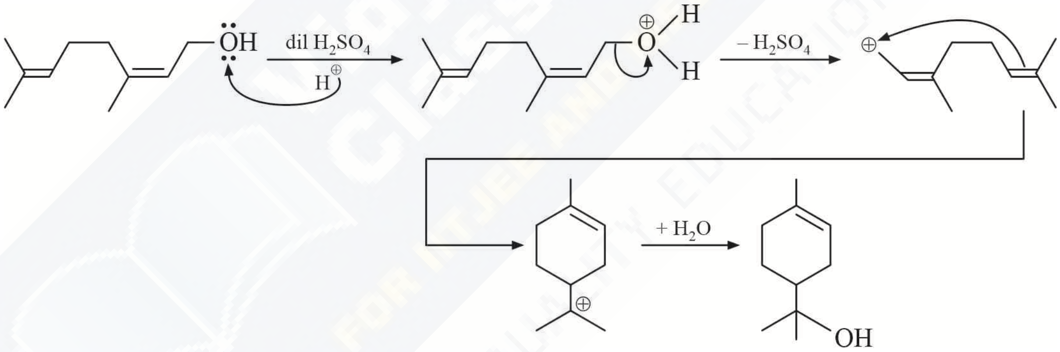
$\rightarrow$ Glucose combines with hydroxylamine to form a monoxime.

14.(3) At a particular temperature as intermolecular forces of attraction increases, vapour presence decrease. So intermolecular forces of attraction of X < Y < Z.

15.(2) The gases that cause green-house effect are —CO_2, CH_4, O_3, chlorofluorocarbon (CFCs) and water vapour.

16.(2) Oxalic acid is a primary standard solution while H_2SO_4 is a secondary standard solution.

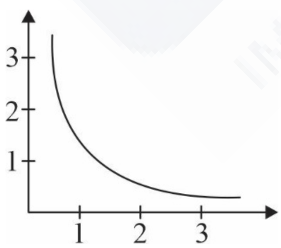
17.(2)



18.(4) Vapours of the liquid with higher boiling point condense before the vapours of the liquid with lower boiling point. Hence isohexane will be distilled out first.

If the difference in boiling points of two liquids is not much, simple distillation cannot be used to separate them.

19.(4)



The correct answer would be

$XY, ksp = 2 \times 10^{-6}\ M^2$

As only this expression of ksp will satisfy all the points on curve.

20.(4) Correct order of 1st ionization energy will be–

Na < Al < Mg < Si

SECTION – 2

21.(–0.93) $2H_2O \longrightarrow O_2 + 4H^+ + 4e^- \quad \in^\circ_{Red} = 1.23\,V$

$$E_{cell} = E^\circ_{cell} - \frac{0.059}{4}\log[H^+]^4$$

$$E_{cell} = -1.23 - \frac{0.059}{4}\log(10^{-5})^4$$

$$= -1.23 - \frac{0.059}{4}\log 10^{-20}$$

$$= -1.23 - \frac{0.059}{4}\times(-20)$$

$$= -1.23 + 0.295$$

$$= -0.935V$$

22.(4.95 to 4.97)

$$\text{Concentration in ppm} = \frac{\text{mass of solute}}{\text{Mass of solution}}\times 10^6$$

$\Rightarrow \quad 10 = \dfrac{\text{mass of Fe (in gm)}}{100\times 10^3\,\text{gm}}\times 10^6$

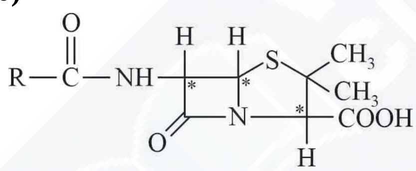
$\Rightarrow$ Mass of Fe = 1 gm

Molecular weight of $FeSO_4.7H_2O = 278$ gm / mole

$\therefore$ 56 gm of Fe is present in 278 gm of salt

56 gm of Fe is present in $\dfrac{278}{56} = 4.96$ gm of salt

23.(3.00)



(Structure of pencillin)

24.(48.00)

Work done = Area under the curve

$= \frac{1}{2}(6+10)\times 6$

= 48 J

25.(26.80 to 27.00)

$$\underset{0.3\text{ gm}}{[Co(NH_3)_6]Cl_3} + \underset{x\text{ ml of }0.125\text{ M}}{3AgNO_3} \longrightarrow 3AgCl + [Co(NH_3)_6](NO_3)_3$$

$$\frac{0.3}{267.46}\times 3 = \frac{x}{1000}\times 0.125$$

$\Rightarrow \quad x = 26.91$ ml

MATHEMATICS

SECTION – 1

1.(1) Rolle's theorem will be applicable if

$f(b) = f(u)$

$$\ln\left(\frac{3^2+\alpha}{7\times 3}\right) = \ln\left(\frac{4^2+\alpha}{7\times 4}\right)$$

$$\frac{9+\alpha}{3} = \frac{16+\alpha}{4}; \qquad \alpha = 12$$

$$f(x) = \ln\left(\frac{x^2+12}{7x}\right) = \ln\left(\frac{x+12/x}{7}\right)$$

$$f'(x) = \frac{7x}{x^2+12}\times\left(\frac{1-\frac{12}{x^2}}{7}\right) = \frac{(x^2-12)}{x(x^2+12)} = 0$$

$$x^2 = 12 \text{ or } c = \sqrt{12}$$

$$f''(x) = (x^2-12)\frac{d}{dx}\left(\frac{1}{x(x^2+12)}\right) + \frac{2x}{x(x^2+12)}; \qquad f''(\sqrt{12}) = \frac{2}{24} = \frac{1}{12}$$

2.(3) $f(x) = \dfrac{2^{1+x}+2^{1-x}}{2} \quad + \quad \dfrac{3^x+3^{-x}}{2}$

↓ ↓

Minimum at $x = 0$ Minimum at x = 0

Hence $f(x)$ is minimum at $x = 0$

$$f(x)_{\min} = \frac{2+2+1+1}{2} = 3$$

3.(4) Volume of parallelepiped $= \left|[\vec{u}\ \vec{v}\ \vec{w}]\right| = 1$

$$\begin{vmatrix} 1 & 1 & \lambda \\ 1 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix} = \pm 1$$

$$(-2+5-\lambda) = \pm 1$$

$$\lambda = 2 \text{ or } 4$$

$$\vec{w} = 2\hat{i} + \hat{j} + \hat{k}$$

$$\cos\theta = \frac{\vec{u}\cdot\vec{w}}{|\vec{u}||\vec{w}|}$$

$\lambda = 2$:

$$\vec{u} = \hat{i} + \hat{j} + 2\hat{k}$$

$$\cos\theta = \frac{2+1+2}{\sqrt{6}\times\sqrt{6}} = \frac{5}{6}$$

$\lambda = 4$:

$$\vec{u} = \hat{i} + \hat{j} + 4\hat{k}$$

$$\cos\theta = \frac{2+1+4}{\sqrt{18}\times\sqrt{6}} = \frac{7}{6\sqrt{3}}$$

4.(1) $\lim\limits_{x\to 0}\left(\dfrac{3x^2+2}{7x^2+2}\right)^{1/x^2}$

$= e^{\lim\limits_{x\to 0}\frac{1}{x^2}\left(\frac{3x^2+2}{7x^2+2}-1\right)}$

$= e^{\lim\limits_{x\to 0}\frac{1}{x^2}\times\frac{-4x^2}{7x^2+2}}$ $\quad = e^{-2} = \dfrac{1}{e^2}$

5.(1) $x^2 + bx + 45 = 0$ → $\alpha + i\beta$, $\alpha - i\beta$

Product of roots $\alpha^2+\beta^2 = 45$, sum of roots $= -b = 2\alpha$

or

We can say z will lies on

$x^2 + y^2 = 45$ …(1)

Also, $|z+1| = 2\sqrt{10}$

$z = x + iy$

$(x+1)^2 + y^2 = 40$

$= x^2 + y^2 + 2x + 1 = 40$ …(2)

Solving (1) and (2)

45 + 2x + 1 = 40

$x = -3$

$b = -2x = 6$

$b^2 - b = 30$

6.(2) $(P \wedge (P \to Q)) \to Q$

$= (P \wedge (\sim P \vee Q)) \to Q$

$= ((\underset{\downarrow F}{P \wedge \sim P}) \cup (P \wedge Q) \to Q$

$= (P \wedge Q) \to Q$

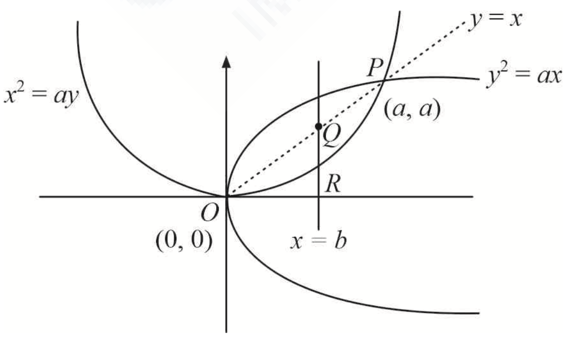
$= (\sim (P \wedge Q)) \vee Q$

$= \ \sim P \vee \sim Q \vee Q$

$= \ \sim P \vee T$

= T (tautology)

7.(3)



$Q(b, b)$

Area $(\Delta OQR) = \frac{1}{2} b \times b = \frac{1}{2}$

$b = 1$

$C_1 : y^2 = ax$

$y = \sqrt{a} \cdot \sqrt{x}$ for $x > 0$

$C_2 : x^2 = ay$

$y = \frac{x^2}{a}$

Area bounded by C_1 and $C_2 = 2.$ Area bounded by C_1 and C_2 in (0, 1)

$$\int_0^a \left(\sqrt{a} \cdot \sqrt{x} - \frac{x^2}{a} \right) dx = 2\int_0^1 \left(\sqrt{a} \cdot \sqrt{x} - \frac{x^2}{a} \right) dx$$

$$\frac{2}{3}\sqrt{a} \cdot a^{3/2} - \frac{a^2}{3} = 2\left\{ \frac{2}{3}\sqrt{a} - \frac{1}{3a} \right\}$$

$$\frac{a^2}{3} = \frac{4}{3}\sqrt{a} - \frac{2}{3a}; \qquad a^3 = 4a\sqrt{a} - 2$$

$$(a^3 + 2)^2 = 16a^3; \qquad a^6 - 12a^3 + 4 = 0$$

8.(3) Mean $(\overline{x}) = 20$ $\therefore$ $x_1 + x_2 + \ldots + x_{10} = 200$

$S.D = 2$

Let observation are $x_1, x_2, x_3 \ldots x_{10}$

New observations are

$px_1 - q,\ px_2 - q, \ldots px_{10} - q$

New mean $= \frac{(px_1 - q) + (px_2 - q) + \ldots + (px_{10} - q)}{10}$

$$= \frac{p(x_1 + x_2 + \ldots + x_{10}) - 10q}{10} \qquad = \frac{200p - 10q}{10}$$

$$= 20p - q = \frac{1}{2}(20)$$

$$20p - q = 10 \qquad \ldots(1)$$

Old S.D. $= \sqrt{\frac{\Sigma(x_i - \overline{x})^2}{10}} = \sqrt{\frac{\Sigma(x_i - 20)^2}{10}}$

New S.D. $= \sqrt{\frac{\Sigma((px_i - q) - (20p - q))^2}{10}}$

$$= \sqrt{\frac{p^2(x_i - 20)^2}{10}} = \frac{1}{2} \text{ old S.D.}$$

$$= \frac{1}{2}\sqrt{\frac{(x_i - 20)^2}{10}}; \qquad \sqrt{p^2} = \frac{1}{2}$$

$$p = \frac{1}{2} \quad \text{or} \quad -\frac{1}{2}$$

$\because$ $20p - q = 10;$ $q = 20p - 10$

$$= 20\left(-\frac{1}{2}\right) - 10 = -20 \quad \text{or} \quad 20\left(\frac{1}{2}\right) - 10 = 0$$

9.(Bonus)

$$\frac{dy}{\sqrt{1-y^2}} = -\frac{dx}{\sqrt{1-x^2}}$$

$$\sin^{-1}(y) = -\sin^{-1}(x) + C$$

$$y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\frac{\pi}{3} = -\frac{\pi}{6} + C \Rightarrow \; C = \frac{\pi}{2}$$

$$\sin^{-1}(y) = -\sin^{-1}(x) + \frac{\pi}{2}$$

$$x = -\frac{1}{\sqrt{2}}$$

$$\sin^{-1}(y) = \frac{\pi}{4} + \frac{\pi}{2}$$ Not Possible

10.(2) $\int \frac{\cos x \, dx}{\sin^3 x(1+\sin^6 x)^{2/3}} = \int \frac{\cos x \, dx}{\sin^3 x \cdot \sin^4 x \, (\cos ec^6 x + 1)^{2/3}}$

$$I = \int \frac{\cos ec^5 x \cdot \cos ecx \cdot \cot x \, dx}{(\cos ec^6 x + 1)^{2/3}}$$

$$1 + \cos ec^6 x = t$$

$$6 \cos ec^5 x \cdot (-\cos ecx \; \cot x) dx = dt$$

$$I = -\int \frac{dt}{6t^{2/3}} = -\frac{3}{6} t^{1/3} = -\frac{1}{2}(1+\cos e^6 x)^{1/3} = -\frac{1}{2} \frac{(1+\sin^6 x)^{1/3}}{\sin^2 x}$$

Therefore $f(x) = -\frac{1}{2} \cos ec^2 x$ and $\lambda = 3$

$\Rightarrow \quad \lambda f(\pi/3) = 3 \times \frac{-1}{2} \times \frac{4}{3} = -2$

11.(1) $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ …(1)

$$\vec{a} = 3\hat{i} + 8\hat{j} + 3\hat{k}$$

$$\vec{p} = 3\hat{i} - \hat{j} + \hat{k}$$

$\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ …(2)

$$\vec{b} = -3\hat{i} - 7\hat{j} + 6\hat{k}$$

$$\vec{q} = -3\hat{i} + 2\hat{j} + 4\hat{k}$$

Shortest distance between lines is $d = \left| \frac{(\vec{a} - \vec{b}).(\vec{p} \times \vec{q})}{\left|\vec{p} \times \vec{q}\right|} \right|$

$$\left| \frac{(6\hat{i} + 15\hat{j} - 3\hat{k}).(-6\hat{i} - 15\hat{j} + 3\hat{k})}{\left|-6\hat{i} - 15\hat{j} + 3\hat{k}\right|} \right| = \left| \frac{-36 - 225 - 9}{\sqrt{36 + 225 + 9}} \right| = \sqrt{270} = 3\sqrt{30}$$

12.(2) $x + 2y + 3z = 1$

$3x + 4y + 5z = \mu$

$4x + 4y + 4z = \delta$

Here $D = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 4 & 4 \end{vmatrix} = 0$

$$D_1 = \begin{vmatrix} 1 & 2 & 3 \\ \mu & 4 & 5 \\ \delta & 4 & 4 \end{vmatrix} = 4\mu - 2\delta - 4 = 2(2\mu - \delta - 2)$$

$$D_2 = \begin{vmatrix} 1 & 1 & 3 \\ 3 & \mu & 5 \\ 4 & \delta & 4 \end{vmatrix} = -8\mu + 4\delta + 8 = -4(2\mu - \delta - 2)$$

$$D_3 = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 4 & \mu \\ 4 & 4 & \delta \end{vmatrix} = 4\mu - 2\delta - 4 = 2(2\mu - \delta - 2)$$

For inconsistent system $D = 0$ and atleast one of D_1, D_2, D_3 should be non zero.

$\Rightarrow \quad 2\mu - \delta - 2 \neq 0 \qquad$ Now check option

13.(2) Because A and B are two independent events therefore.

$$P(A'/B') = P(A') = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(A/B') = P(A) = \frac{1}{3}; \qquad P(A/B) = P(A) = \frac{1}{3}$$

14.(4) $A(1,-1), B(0,2), P(x', y')$

Area $(\Delta PAB) = 5$

$$\frac{1}{2}\begin{vmatrix} x' & y' & 1 \\ 1 & -1 & 1 \\ 0 & 2 & 1 \end{vmatrix} = \pm 5$$

$$\frac{1}{2}[x'(-3) - y'(1) + 1(2)] = \pm 5$$

$-3x' - y' + 2 = \pm 10; \qquad -3x' - y' = 8$ or -12

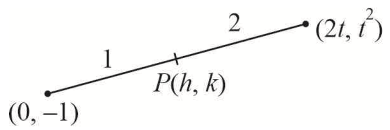
Also, $3x' + y' = 4\lambda$

By solving we get $\lambda = -2, 3$

15.(4) Let a point on the parabola is $(2t, t^2)$

$\Rightarrow \quad h = \dfrac{2t}{3} \quad \Rightarrow \quad t = \dfrac{3h}{2} \qquad \ldots(1)$

$k = \dfrac{t^2 - 2}{3} \qquad \ldots(2)$



From (1) and (2)

$$3k + 2 = \frac{9h^2}{4}$$

Locus of $P(h, k)$ is $\quad 9x^2 = 12y + 8$

16.(4) $f(x)=\frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}}=\frac{y}{1}$

Using componendo – Dividendo

$$\frac{y+1}{y-1}=\frac{2.8^{2x}}{-2.8^{-2x}}$$

$\Rightarrow \quad 8^{4x}=\frac{1+y}{1-y} \qquad \Rightarrow \quad 4x=\log_8\left(\frac{1+y}{1-y}\right)$

$x=\frac{1}{4}\log_8\left(\frac{1+y}{1-y}\right)=f^{-1}(y) \qquad \Rightarrow \quad f^{-1}(x)=\frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right)$

17.(2) $f(x)=x\cos^{-1}(-\sin|x|)$

For $x\in\left(0,\frac{\pi}{2}\right)$

$\Rightarrow \quad f(x)=y=x\cos^{-1}(-\sin x)$

$=x[\pi-\cos^{-1}\sin x]$

$=x\left[\pi-\left(\frac{\pi}{2}-\sin^{-1}\sin x\right)\right]$

$=x\left[\frac{\pi}{2}+\sin^{-1}\sin x\right]$

$y=x\left[\frac{\pi}{2}+x\right]$

$y'=\frac{\pi}{2}+2x>0$ $f(x)$ is increasing

$y''=2>0 \quad \Rightarrow \quad f'(x)$ is increasing

For $x\in\left(-\frac{\pi}{2},0\right)$

$f(x) \quad =x\cos^{-1}(+\sin x)$

$=x\left[\frac{\pi}{2}-\sin^{-1}\sin x\right]$

$=x\left[\frac{\pi}{2}-x\right]$

$f'(x)=\frac{\pi}{2}-2x>0$

$f''(x)=-2<0$

$\therefore \quad f(x)$ is increasing $\qquad \therefore \quad f'(x)$ is decreasing

18.(3) $\frac{dy}{dx}=\frac{1}{2}\frac{d}{dx}(\sin^{-1}f(x))$

$2y=\sin^{-1}(f(x))+C,\ y(\sqrt{3})=\frac{\pi}{6} \quad \Rightarrow \quad C=0$

$y(-\sqrt{3})=-\frac{\pi}{6}$

19.(4) Ellipse $\frac{x^2}{1/2}+\frac{y^2}{1}=1$

Let $P\left(\frac{1}{\sqrt{2}}\cos\theta, \sin\theta\right)$

Also P satisfies $y = mx$

$$\sin\theta = m\left(\frac{1}{\sqrt{2}}\cos\theta\right)$$

$$\Rightarrow \quad \tan\theta = \frac{m}{\sqrt{2}} \qquad \ldots(1)$$

Equation of normal at P

$$\frac{\frac{1}{\sqrt{2}}x}{\cos\theta}-\frac{y}{\sin\theta}=\frac{1}{2}-1$$

It satisfies $\left(-\frac{1}{3\sqrt{2}},0\right)$ and $(0, \beta)$

$$\Rightarrow \quad -\frac{1}{6\cos\theta}=-\frac{1}{2} \text{ and } -\frac{\beta}{\sin\theta}=-\frac{1}{2} \quad \Rightarrow \quad \cos\theta=\frac{1}{3} \qquad \sin\theta = 2\beta$$

$$\Rightarrow \quad \tan\theta = \frac{\sin\theta}{\cos\theta}=\frac{2\beta}{1/3}=6\beta \qquad \ldots(2)$$

From (1) and (2)

$$\frac{m}{\sqrt{2}}=6\beta$$

Also if $\cos\theta=\frac{1}{3} \Rightarrow \tan\theta = 2\sqrt{2} \Rightarrow 6\beta = 2\sqrt{2}$; $\beta=\frac{\sqrt{2}}{3}$

20.(2) $a={}^{19}C_9=\frac{19!}{9!\ 10!}$

$b={}^{20}C_{10}=\frac{20!}{10!\ 10!}=\frac{20\times 19!}{10\times 9!\times 10!}$

$c={}^{21}C_{10}=\frac{21!}{11!\ 10!}=\frac{21\times 20\times 19!}{11\times 10\times 9!\times 10!}$

$$\Rightarrow \quad \frac{a}{1}=\frac{b\times 10}{20}=\frac{c\times 11\times 10}{21\times 20} \Rightarrow \quad \frac{a}{1}=\frac{b}{2}=\frac{c\times 11}{42} \Rightarrow \quad \frac{a}{11}=\frac{b}{22}=\frac{c}{42}$$

SECTION – 2

21.(8) $2x^2+(a-10)x+\frac{33}{2}=2a$

For real roots $D\ge 0$

$(a-10)^2-4\times 2\times\left(\frac{33}{2}-2a\right)\ge 0$

$a^2+100-20a-132+16a\ge 0$

$a^2-4a-32\ge 0$

$(a-8)(a+4)\ge 0$

$a\in(-\infty,-4]\cup[8,\infty)$

$a_{\text{least}}=8$

22.(672) Sum of diagonal elements of $AA^T = 3 = a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{21}^2 + a_{22}^2 + a_{23}^2 + a_{31}^2 + a_{32}^2 + a_{33}^2$

Therefore out of these 9 terms, 6 terms should be zero and remaining 3 terms are either +1 or –1

$\Rightarrow$ Number of total possible matrices $= {}^9C_6 \times 2 \times 2 \times 2 = 672$

23.(4) Let $P(x_1 y_1)$

$\Rightarrow \quad y_1^2 - 3x_1^2 + y_1 + 10 = 0 \qquad \ldots(1)$

$y^2 - 3x^2 + y + 10 = 0$

Differentiate curve with respect to x

$$2y\frac{dy}{dx} - 6x + \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{6x}{2y+1}$$

Slope of normal at $P(x_1 y_1) = -\frac{(2y_1+1)}{6x_1}$

Equation of normal at $P(x_1, y_1)$

$$y - y_1 = -\frac{(2y_1+1)}{6x_1}(x - x_1)$$

It satisfies (0, 3/2)

$\Rightarrow \quad \frac{3}{2} - y_1 = \frac{2y_1+1}{6}$

$\Rightarrow \quad 9 - 6y_1 = 2y_1 + 1$

$y_1 = 1$

From equation $x_1 = \pm 2$

Slope of tangent at $P(x_1, y_1) = \frac{6x_1}{2y_1+1} = \pm 4$

$|m| = 4$

24.(490) 5 red, 4 black, 3 white

There are 4 possible cases

(1) 0 red + 4 other $= {}^7C_4 = 35$

(2) 1 red + 3 other $= {}^5C_1 \times {}^7C_3 = 175$

(3) 2 red + 2 other $= {}^5C_2 \times {}^7C_2 = 210$

(4) 3 red + 1 other $= {}^5C_3 \times {}^7C_1 = 70$

Total ways = 490

25.(1540) $\sum_{k=1}^{20}(1+2+3+\ldots+k)$

$$\frac{1}{2}\sum_{k=1}^{20} k(k+1) = \frac{1}{2}\sum_{k=1}^{20}k^2 + \frac{1}{2}\sum_{k=1}^{20}k = \frac{1}{2}\times\frac{20\times 21\times 41}{6} + \frac{1}{2}\times\frac{20\times 21}{2} = 1540$$