# SOLUTIONS

## JEE Main – 2020 | 9th January 2020 (Evening)

## PHYSICS

### SECTION – 1

**1.(1)** $I = \dfrac{V}{R(net)} = \dfrac{9}{(10+5+10+5)} = \dfrac{9}{30} = 0.3A$

**2.(Bonus)**

$A_1 + B_1 + C_1 = 280.6324 = 280.6$ (Rounded off to 1 decimal)

$A_2 + B_2 + C_2 = 280.722 = 280.7$ (Rounded off to 1 decimal)

$A_3 + B_3 + C_3 = 280.6642 = 280.7$ (Rounded off to 1 decimal)

$A_4 + B_4 + C_4 = 280.691 = 281$ (Rounded off to 0 decimal)

(In addition or subtraction, the final result should retain as many decimal places as are there in the number with least decimal places.) No option gives correct relation

**3.(3)** $I = \dfrac{V}{X} = \dfrac{10}{\left[340\times 40\times 10^{-3} - \dfrac{1}{314\times 10^{-4}}\right]} = \dfrac{10}{|13.6-31.84|} = \dfrac{10}{(18.25)} = \dfrac{10}{18.25} = 0.55$

$I \simeq .52\cos(314t)$

**4.(3)** Mean free time $= \dfrac{\text{mean free path}}{V_{rms}} = \dfrac{1}{\sqrt{2}\pi d^2}\dfrac{\sqrt{M}}{\sqrt{3RT}}$

$\dfrac{\tau_1}{\tau_2} = \sqrt{\dfrac{M_1}{M_2}}\left(\dfrac{d_2}{d_1}\right)^2 = \sqrt{\dfrac{40}{10}}\left(\dfrac{0.1}{0.07}\right)^2 = \sqrt{\dfrac{2}{7}}\times\dfrac{100}{49} = 1.09$

No option matches. But closest is 1.83

**5.(1)** $\dfrac{1}{\lambda} = Rz^2\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$

$\dfrac{1}{\lambda} = R\times 9\left[\dfrac{1}{1} - \dfrac{1}{9}\right] = 9R\left[\dfrac{8}{9}\right] = 8R = 8\times 1.1\times 10^7$

$\Rightarrow \quad \lambda = \dfrac{1\times 10^{-7}}{8\times 1.1} = \dfrac{100\times 10^{-9}}{8.8} \cong 11.4\ nm$

**6.(2)** $2\pi rT + \dfrac{2}{3}\pi r^3\rho g = \dfrac{4}{3}\pi r^3 d\, g$

$2\pi rT = \dfrac{2}{3}\pi r^3[2d-\rho]g \Rightarrow \sqrt{\dfrac{3T}{(2d-\rho)g}} = r$

**7.(3)** At the topmost point velocity before collision = $u\cos\dfrac{\pi}{3}$

by conservation of linear momentum
$mu\cos\pi/3 + mu = 2mv$

$$v = \frac{3u}{4}$$

Height = $\frac{u^2 \sin^2 \pi/3}{2g}$ ; $H = \frac{3u^2}{8g}$

Range = $v\sqrt{\frac{2H}{g}} = \frac{3u}{4}\sqrt{\frac{2}{g}\left(\frac{3u^2}{8g}\right)} = \frac{3\sqrt{3}}{8}\frac{u^2}{g}$

**8.(3)** $\Delta l = \left[\frac{Fl}{Ay}\right] \Rightarrow \frac{\Delta \ell_1}{\Delta \ell_2} = \frac{A_2}{A_1}$

Energy density = $\frac{1}{2}\times$stress $\times$strin

$$\frac{E_1}{E_2} = \frac{F}{A_1}\frac{A_2}{F}\frac{\Delta\ell_1}{L}\frac{L}{\Delta\ell_2}$$

$$\frac{1}{4} = \frac{A_2}{A_1}\frac{\Delta\ell_1}{\Delta\ell_2}; \quad \frac{A_2}{A_1} = \frac{1}{2}$$

**9.(3)** $X_{cm} = \frac{\int x\, dm}{\int dm} = \frac{\int_0^L x\left(a + b\frac{x^2}{L^2}\right)dx}{\int_0^L \left(a + b\frac{x^2}{L^2}\right)dx}$

$$X_{cm} = \frac{\left[a\frac{x^2}{2} + \frac{bx^4}{4L^2}\right]_0^L}{\left[ax + \frac{bx^3}{3L^2}\right]_0^L} = \frac{\left[a\frac{L^2}{2} + \frac{b}{4}L^2\right]}{\left[aL + \frac{b}{3}L\right]} = \frac{\frac{aL}{2} + \frac{bL}{4}}{\left[a + \frac{b}{3}\right]} = \frac{\left(\frac{2a+b}{4}\right)L}{\left[\frac{3a+b}{3}\right]}$$

$$X_{am} = \frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$$

**10.(3)** $v_e = \sqrt{\frac{GM}{R}} = \frac{v_{e_1}}{v_{e_2}} = \frac{1}{1} \Rightarrow n = 4$

**11.(4)** $\frac{f_n}{f_{n+1}} = \frac{420}{496} = \frac{6}{7}$

$$420 = \frac{6}{2L}\sqrt{\frac{T}{\mu}} = \frac{6}{2\times L}\sqrt{\frac{540}{6\times 10^{-3}}} \;;\; 420 = \frac{3}{L}\sqrt{9\times 10^4} = \frac{3\times 3\times 10^2}{L} \;;\; L = \frac{96}{42} = 2.1(m)$$

**12.(4)** $\frac{mv^2}{\frac{r}{2}} = qvB = evB$

$$\frac{2mv}{r} = eB; \qquad v = \left(\frac{eBr}{2m}\right) = \left[\frac{e\mu_0 nIR}{2m}\right]$$

**13.(1)** $m\omega^2(l + x) = kx$

$$x = \frac{m\omega^2 l}{k - m\omega^2}$$

$$\frac{x}{l} = \frac{m\omega^2}{k} \qquad (k >> m\omega^2)$$

**14.(1)** $\lambda = \frac{h}{mv}$

$v = 0 + at = at = \left(\frac{eE}{m}\right)t$

$\lambda = \frac{h}{m\,at} = \frac{h\,m}{m\,eE\,t}$

$\lambda = \frac{h}{e\,E\,t}$

$\frac{dy}{dt} = \frac{-h}{eE}\frac{1}{t^2}$

**15.(3)** $\tau = MB\sin\theta = MB\theta = (iAB)\theta$

$\tau = k\theta \;\; k = (i\,A\,B)$

$T = 2\pi\sqrt{\frac{I}{k}} = 2\pi\sqrt{\frac{MR^2}{2(i\pi R^2)B}} = \left(\frac{4\pi^2\,MR^2}{2i\pi R^2 B}\right)^{\frac{1}{2}}$ ; $T = \sqrt{\frac{2\pi M}{iB}}$

**16.(4)** B is $\perp$ to direction of propagation of e.m wave $B = B_0\left(\frac{\hat{i} - j}{\sqrt{2}}\right)\cos(\omega t - kr)$

**17.(3)** Loss in G.P.E of $m_1$ = Gain in K.E. of blocks

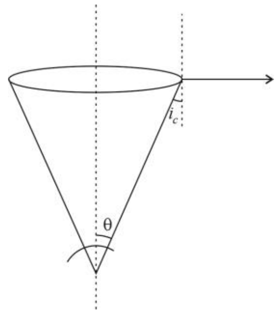
+ Gain in G.P.E of $m_2$ + Gain in rotational K.E. of cylinder

$m_2gh - m_1gh = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I_{am}\omega^2$

$(m_2 - m_1)gh = \frac{1}{2}m_1(\omega R)^2 + \frac{1}{2}m_2(\omega R)^2 + \frac{1}{2}I\omega^2$

$(m_2 - m_1)gh = \frac{1}{2}\left(m_1R^2 + m_2R^2 + I\right)\omega^2$ ; $\omega = \sqrt{\frac{2(m_2 - m_1)gh}{\left[(m_1 + m_2)R^2 + I\right]}}$

**18.(3)** Light flux = $\frac{\phi_0}{2}[1 - \cos\theta] = \frac{\phi_0}{2}\left[1 - \frac{\sqrt{7}}{4}\right] = \frac{\phi_0}{2}[0.34] = 0.17$




$i_c = \theta =$ critical angle $\Rightarrow 17\%$

$\sin\theta = \sin i_c = \frac{3}{4}$

$\cos\theta = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4} = \frac{2.645}{4} = 0.661$

**19.(4)** As acceleration is constant $S_y = u_yt + \frac{1}{2}a_yt^2$

$32 = \frac{1}{2} \times 4 \times t^2$ ; $t = 4\text{s}$

$S_x = u_xt + \frac{1}{2}a_xt^2 = 3 \times 4 + \frac{1}{2} \times 6 \times 16 = 12 + 48$ ; $S_x = 60\,\text{m}$

**20.(1)** $Q_A = VC$ open circuit due to reverse biased diode

$Q_B = \frac{VC}{e} = VCe^{-t/RC} = \frac{VC}{e}$ due to forward biased diode

**SECTION – 2**

**21.(1818,1819)**

$$T_1V_1^{4-1} = T_2V_2^{4-1} \Rightarrow T_2 = T_1\left(\frac{V_1}{V_2}\right)^{1.4-1} = T_1(16)^{2/5}$$

$$T_3 = 2T_2 = 2(300)(16)^{2/5} = 1818.8$$

**22.(750)** $n\lambda\frac{D}{d} = 15\times 500\times\frac{D}{d} = 10\times\lambda\times\frac{D}{d}$

$750\ nm = \lambda$

**23.(40)** Potential difference across resistors = 12 – 8 = 4V

$$I = \frac{4}{400} = \frac{1}{100}A$$

P.d across each Zener diode $= \frac{8}{2} = 4V$

Power dissipated $= VI = (4)\left(\frac{1}{100}\right) = 40\times 10^{-3}\omega\ = 40m\omega$

**24.(–48)** $\vec{E} = 4x\hat{j} - (y^2+1)\hat{j}$

For ABCD: $\overrightarrow{dA} = (dA)\vec{K} \Rightarrow \vec{E}\cdot\overrightarrow{dA} = 0$

$\therefore\quad \phi_1 = 0$

For BCGF : $\overrightarrow{dA} = (dA)\hat{j}$

$\therefore\quad \phi_2 = \int[4x\hat{i} - (y^2+1)\hat{j}].(dA\hat{i})$

$= \int 4x dA = \int 4(3)dA = 12A$

$= 12(4) = 48$

$\phi_1 - \phi_2 = -48$

**25.(40)** $\frac{R}{S} = \frac{l}{100-l}\qquad \frac{R}{S} = \frac{25}{75} \Rightarrow R = \frac{S}{3}$

$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi d^2}\ ;\quad R' = \frac{\rho\frac{l}{2}}{\frac{\pi d^2}{4}} = 2\frac{\rho l}{\pi d^2} = 2R = \frac{2S}{3}$$

$$\frac{R'}{S} = \frac{l}{100-l}\ ;\quad \frac{2}{3} = \frac{l}{100-l}\ ;\quad 200 - 2l = 3l\ ;\quad 200 = 5l\quad ;\quad 40 = l$$

## CHEMISTRY

**SECTION – 1**

**1.(2)** The amount of oxygen required by bacteria to break down the inorganic matter present in a certain volume of a sample of water is called biochemical oxygen demand (BOD)

**2.(1)** $Zn + 2NaOH \rightarrow Na_2ZnO_2 + H_2$

$Zn + 2HCl \rightarrow ZnCl_2 + H_2$

According to stoichiometry in both the reactions, equal number of mole of $H_2$ are evolved

**3.(1)** $Ca(OH)_3 \rightarrow \underset{S}{Ca^{3+}} + 3\underset{3S}{OH^-}$

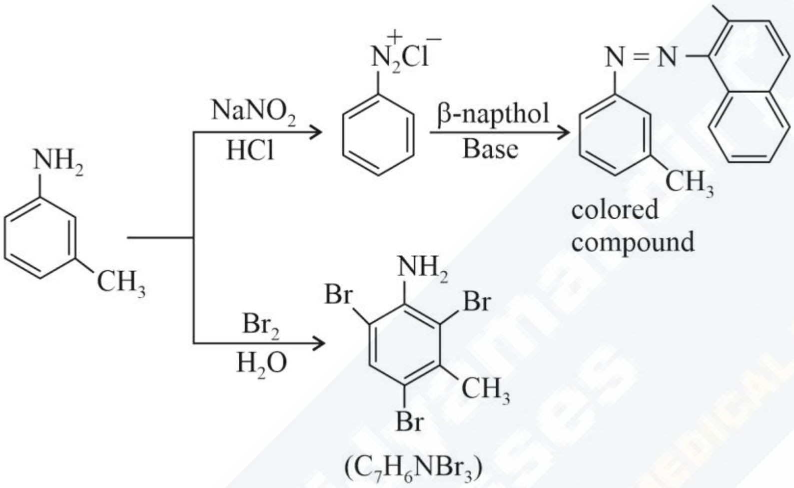
$K_{sp} = (s)(3s)^3$

$6\times10^{-31} = 27s^{-4}$; $\qquad S = \left(\frac{6}{27}\times10^{-31}\right)^{\frac{1}{4}}$

$\left[OH^-\right] = 3s = 3\left(\frac{6}{27}\times10^{-31}\right)^{\frac{1}{4}} = \left(18\times10^{-31}\right)^{\frac{1}{4}}$ M

**4.(4)** In Benzene, total six $sp^2$ hybrid carbon atoms are present. Each carbon atom has 3 $sp^2$ hybrid orbitals. Therefore, total $sp^2$ hybrid orbital are 18 in Benzene.

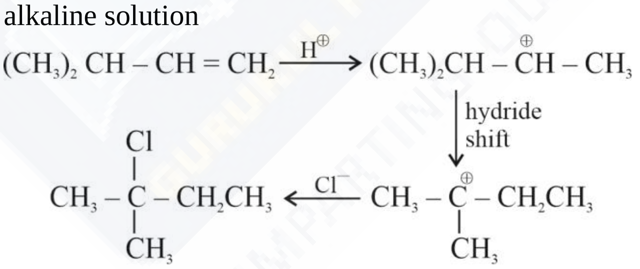
**5.(2)**




**6.(4)** $\Delta S = \int \frac{dq}{T}$; $S = \int_0^T \frac{ncdT}{T}$

**7.(4)** All carbohydrates – Monosaccharides, disaccharides, and polysaccharides should give a positive reaction.
Barfoed's test detects monosccharides. It is based on reduction of copper (II) acetate to copper (I) oxide which forms brick red precipitate.
Biuret test detects presence of peptide bonds. Copper (II) ion forms mauve colored complexes in an alkaline solution

**8.(3)**

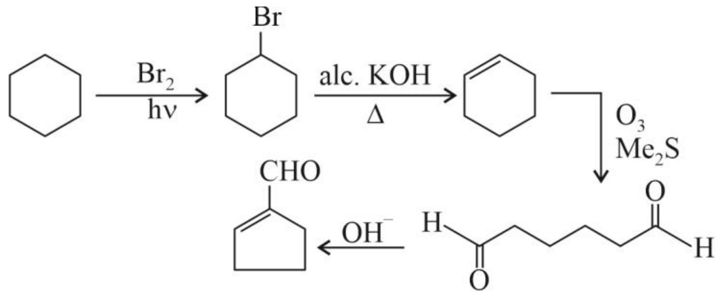



In this reaction, major product is chiral

**9.(1)** $B_3N_3H_3Cl_3 + LiBH_4 \rightarrow B_3N_3H_6 + LiCl + BCl_3$
(A)
$B_3N_3H_3Cl_3 + 3MeMgBr \rightarrow B_3N_3H_3(CH_3)_3 + 3MgBrCl$
(A) (C)

**10.(3)** Resonance form of $Cl - CH = CH - NO_2$ is more stable than resonance form of any other given compounds. Hence double bond character in $C - Cl$ bond is maximum and bond length is minimum

**11.(2)**




**12.(3)** Distilled water show least conductivity due to less number of ions to flow in the solution

**13.(4)** Complex (I) $\Rightarrow$ $Cr^{2+}$ $\Rightarrow$ weak field ligand

$t_{2g}^{3}\ \ eg^{1}$

$\mu_s = \sqrt{24}$ B.M

Complex (II) $\Rightarrow$ $Fe^{2+}$ $\Rightarrow$ strong field ligand

$t_{2g}^{6}\ eg^{0}$ ; $\mu_s = 0$

Complex (III) $\Rightarrow$ $Fe^{3+}$ $\Rightarrow$ strong field ligand

$t_{2g}^{5}\ eg^{0}$ ; $\mu_s = \sqrt{3}$ B.M

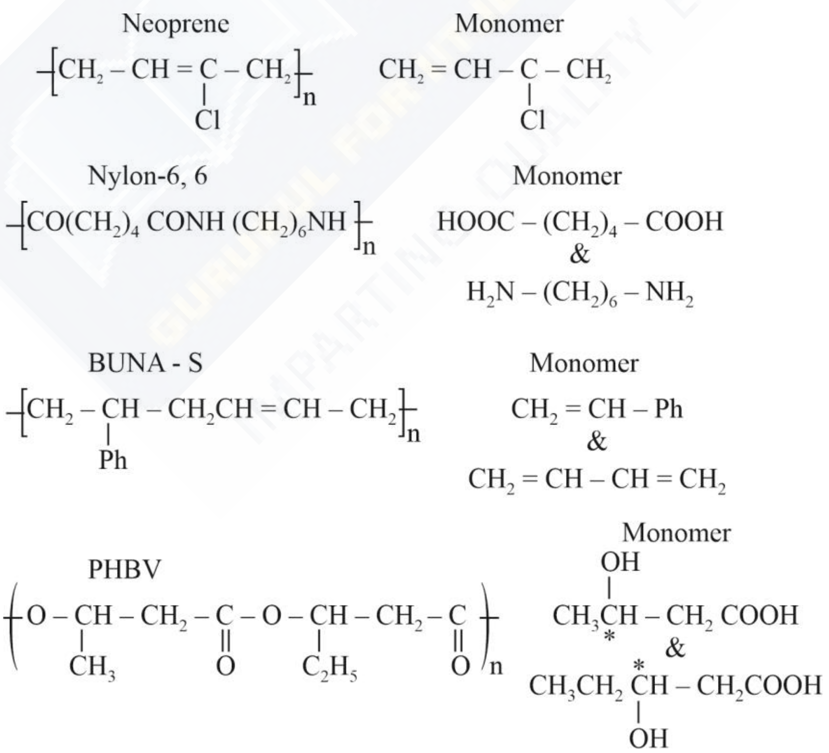
Complex (IV) $\Rightarrow$ $CO^{2+}$ $\Rightarrow$ weak field ligand

$t_{2g}^{5}\ eg^{2}$

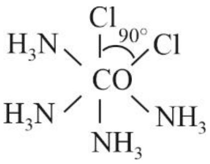
$\mu_s = \sqrt{15}$ B.M

**14.(1)** $k_{eq} = \dfrac{[P]}{[R]} = \dfrac{11}{6} = 1.83 \simeq 2$

**15.(3)**



**16.(3)**



**17.(3)** Statements (a), (c) & (d) are correct.

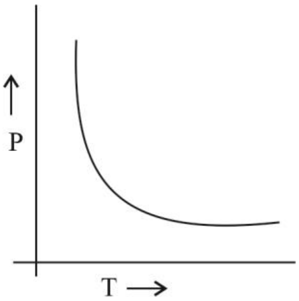
Option (a): Because of small size lithium has very high hydration energy.

Option (b): LiCl is soluble in pyridine because of covalent character. (Incorrect statement)

Option (c): Lithium unlike other alkali metals forms no ethynide on reaction with ethyne.

Option (d): Lithium and magnesium react slowly with water because of high enthalpy of atomisation.

**18.(1)** Theory based



**19.(1)** Basic strength depends upon availability of lone pairs. Greater the resonance of lone pairs lesser is the basic strength.

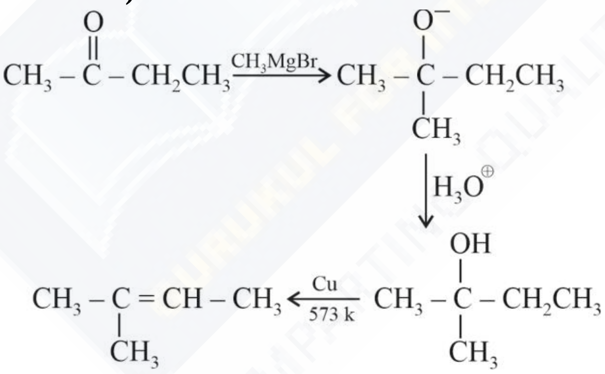
**20.(4)** According to given data of I.E, the element must belong to group I and is monovalent & form hydroxide of type M(OH)

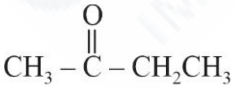
$$\underset{1\,\text{mole}}{M(OH)} + \underset{1\text{ mole}}{HCl} \rightarrow MCl + H_2O$$

$$\underset{1\text{ mole}}{2MOH} + \underset{\frac{1}{2}\text{mole}}{H_2SO_4} \rightarrow M_2SO_4 + H_2O$$

**SECTION – 2**

**21.(66.66 to 66.67)**



A $\Rightarrow$ 

$$\%\text{ carbon} = \left(\frac{12\times 4}{12\times 4+8+16}\right)\times 100 = 66.67$$

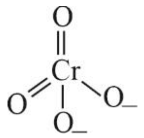
**22.( 3.98 to 3.99)**

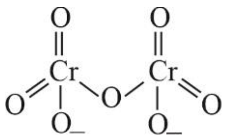
$$\ln\frac{k_2}{k_1} = \frac{Ea}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\ln\left(\frac{60}{40}\right) = \frac{Ea}{8.314}\times\frac{100}{400\times 300}$$

$$Ea = \ln\left(\frac{3}{2}\right)\times 8.314\times 1200 = 3984\text{ J/mol} = 3.984\text{ kJ/mol}$$

**23.(10)** ppm $= \dfrac{10.3\times10^{-3}}{1030}\times10^{6} = 10$

**24.(12)** Chromate ion 

Dichromate ion 

Total number of Cr & O bonds is 12.

**25.(2.18)** $k_f = 2.0$

m = 0.5 m

$\Delta T_f = k_f \times m = 0.5\times 2$

$T_{inital}$ = 272 K

n = 0.1 mol

V = 1 $atm^3$

$P_{gas} = \dfrac{nRT}{v} = \dfrac{0.1\times0.08\times272}{1}$ = 2.176 atm

After releasing piston $P_1V_1 = P_2V_2$, $2.176\times1 = 1\times V_2$

$V_2 = 2.176\ dm^3 \simeq 2.18 dm^3$

## MATHEMATICS

### SECTION – 1

**1.(4)** $$f(x)=\begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

Apply $R_1 \Rightarrow R_1 + R_3 - 2R_2$

$$\Rightarrow \quad f(x)=\begin{vmatrix} 1 & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix} = (x+3)^2-(x+2)(x+4)=1$$

$\therefore \quad f(x)=1 \Rightarrow f(50)=1$

**2.(Bonus)** $x = 2\sin\theta - \sin 2\theta$

$y = 2\cos\theta - \cos 2\theta$

$\dfrac{dx}{d\theta} = +2\cos\theta - 2cos2\theta$

$\dfrac{dy}{d\theta} = -2\sin\theta + 2\sin 2\theta$

$\dfrac{dy}{dx} = \dfrac{\sin 2\theta - \sin\theta}{\cos\theta - \cos 2\theta} = \dfrac{2\cos(3\theta/2)\sin\theta/2}{2\sin(3\theta/2)\sin(\theta/2)}$

$\dfrac{dy}{dx} = \cot\left(\dfrac{3\theta}{2}\right)$

$$\frac{d^2y}{dx^2} = -\cos ec^2\left(\frac{3\theta}{2}\right).\frac{3}{2}\frac{d\theta}{dx}$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=\pi} = -1\times\frac{3}{2}\times\left(\frac{-1}{4}\right) = \frac{3}{8}$$

**3.(1)** $T_{r+1} = {}^{16}C_r\left(\frac{x}{\cos\theta}\right)^{16-r}\left(\frac{1}{x\sin\theta}\right)^r$

Put $16-2r=0 \Rightarrow r=8$

$\therefore$ $T_9$ is independent of $x$

$$T_9 = {}^{16}C_8\,\frac{1}{\sin^8\theta\cos^8\theta} = {}^{16}C_8\,\frac{2^8}{(\sin 2\theta)^8}$$

When $\theta\in\left[\frac{\pi}{8},\frac{\pi}{4}\right]$, $L_1 = {}^{16}C_8\,2^8$ at $\theta=\frac{\pi}{4}$

When $\theta\in\left[\frac{\pi}{16},\frac{\pi}{8}\right]$, $L_2 = {}^{16}C_8\,\frac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8}$ at $\theta=\frac{\pi}{8}$ $= {}^{16}C_8\,2^8.\,2^4$

$$\frac{L_2}{L_1} = \frac{{}^{16}C_8\,2^8.2^4}{{}^{16}C_8\,2^8} = 16$$

**4.(1)** $\left(\frac{1}{2},-2\right)\equiv\left(2t^2,4t\right)\Rightarrow t=\frac{-1}{2}$

$\therefore$ Other end of focal chord is at $t = 2$

$B\left(2(2)^2, 4(2)\right)\equiv B(8,8)$ equation of tangent at B is

$2y = x+2(2)^2 \Rightarrow 2y-x=8$

**5.(1)** $\int\frac{\sec^2\theta}{\sec 2\theta+\tan 2\theta}\,d\theta = \int\frac{\sec^2\theta}{\frac{1+\tan^2\theta}{1-\tan^2\theta}+\frac{2\tan\theta}{1-\tan^2\theta}}\,d\theta = \int\frac{\sec^2\theta(1-\tan\theta)}{1+\tan\theta}\,d\theta$

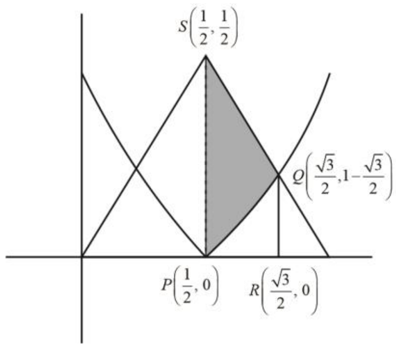
Put $\tan\theta = t$

$\Rightarrow$ $\sec^2\theta\,d\theta = dt$

$\int\frac{1-t}{1+t}dt = \int\left(-1+\frac{2}{1+t}\right)dt = -t+2\log(1+t)+C = -\tan\theta+2\log(1+\tan\theta)+C$

$\Rightarrow$ $\lambda=-1$ and $f(\theta)=1+\tan\theta$

**6.(4)** Refer to the figure




Required area = Are of trapezium PRQS $-\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}\left(x-\frac{1}{2}\right)^2 dx$

$$= \frac{1}{2}\left(\frac{\sqrt{3}-1}{2}\right)\left(\frac{1}{2}+1-\frac{\sqrt{3}}{2}\right)-\frac{1}{3}\left[\left(x-\frac{1}{2}\right)^3\right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{4}-\frac{1}{3}$$

**7.(3)** Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$\therefore \quad v + \frac{xdv}{dx} = \frac{vx^2}{x^2 + v^2x^2} = \frac{v}{1+v^2}$

$\Rightarrow \quad \frac{1+v^2}{v^3}dv = \frac{-1}{x}dx \quad \Rightarrow \quad \left(\frac{1}{v^3} + \frac{1}{v}\right)dv = -\frac{1}{x}dx \quad \Rightarrow \int\left(\frac{1}{v^3} + \frac{1}{v}\right)dv = \int\frac{-1}{x}dx$

$\Rightarrow \quad \frac{-1}{2}\frac{1}{v^2} + \log_e v = -\log_e x + C \quad \Rightarrow \quad \frac{-x^2}{2y^2} = -\log_e y + C$

Put $x = 1$, $y = 1$

$\therefore \quad$ we get $C = -\frac{1}{2}$

$\Rightarrow \quad -x^2 = 2y^2\left(-\log_e y - \frac{1}{2}\right) \quad \Rightarrow \quad x^2 = y^2(1 + 2\log_e y)$

Put $y = e$

$\therefore \quad x^2 = e^2(3) \Rightarrow x = \pm\sqrt{3}e \quad \Rightarrow x = \sqrt{3}e$

**8.(3)** $F'(x) = x^2 g(x)$

$\therefore \quad F'(1) = g(1) = 0 \quad \ldots$(i) $\quad \left[\because g(1) = \int_1^1 f(t)dt = 0\right]$

Now, $\quad F''(x) = 2xg(x) + x^2 f(x)$

$\Rightarrow \quad F''(1) = 0 + 3 = 3 > 0$

$\therefore \quad F(x)$ has a local minima at $x = 1$

**9.(3)** $\sum P(x) = 6K^2 + 5K = 1$

$\Rightarrow \quad K = -1, \frac{1}{6}$

$K = -1$ (rejected) $\left(\because P(x) >, 0\right) \quad \therefore \quad K = \frac{1}{6}$

Now $P(x > 2) = 5K^2 + 3K = \frac{3}{6} + \frac{5}{36} = \frac{23}{36}$

**10.(4)** $A = \{x;\ x \in (-2, 2)\}$

$B = \{x;\ x \in (-\infty, -1] \cup [5, \infty)\}$

$A \cap B = (-2, -1]$

$A \cup B = (-\infty, 2) \cup [5, \infty)$

$A - B = (-1, 2)$

$B - A = (-\infty, -2] \cup [5, \infty)$

**11.(3)** $f(g(x)) = x \quad \Rightarrow \quad f'(g(x))\, g'(x) = 1$

Putting $x = a$, we get $f'(g(a))\, g'(a) = 1$

$\Rightarrow \quad f'(b) \times 5 = 1 \Rightarrow f'(b) = \frac{1}{5}$

**12.(1)** Let G.P is $a, ar, ar^2, \ldots\ldots\ldots$

$$\sum_{n=1}^{100} a_{2n+1} = ar^2 + ar^4 + \ldots\ldots ar^{200} = 200$$

$$\Rightarrow \quad ar^2 \frac{\left(r^{200}-1\right)}{r^2-1} = 200 \qquad \ldots\text{(i)}$$

$$\sum_{n=1}^{100} a_{2n} = ar + ar^3 + \ldots.\ ar^{199} = 100$$

$$\Rightarrow \quad ar \frac{\left(r^{200}-1\right)}{r^2-1} = 100 \qquad \ldots\text{(ii)}$$

Dividing (i) & (ii), we get

$r = 2$

adding we get ,

$a_2 + a_3 + \ldots\ a_{200} + a_{201} = 300$

$$\Rightarrow \quad r\left(a_1 + a_2 + \ldots a_{200}\right) = 300 \;\Rightarrow\; \sum_{n=1}^{200} a_n = \frac{300}{r} = \frac{300}{2} = 150$$

**13.(1)** We have $\begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix} = 7(-20) + 6(20) + 2(10) = 0$

$\therefore$ so infinite solution exists

Now, Equation (i) + 3 eq (iii), we get

$10x - 20z = 0 \quad \Rightarrow \quad x = 2z$

**14.(2)** $x = \dfrac{1}{1-\left(-\tan^2\theta\right)} = \cos^2\theta$ $\qquad \ldots$(i)

$$y = \frac{1}{1-\cos^2\theta} = \frac{1}{\sin^2\theta} \Rightarrow \frac{1}{y} = \sin^2\theta$$

By (1) & (2)

$$\therefore \quad 1 - x = \frac{1}{y} = \sin^2\theta \quad \Rightarrow \quad y(1-x) = 1$$

**15.(4)** $\lim\limits_{x\to 0} x\left[\dfrac{4}{x}\right] = A$

$$\Rightarrow \quad \lim_{x\to 0} x\left(\frac{4}{x} - \left\{\frac{4}{x}\right\}\right) = A \Rightarrow \lim_{x\to 0} x\left(4 - x\left\{\frac{4}{x}\right\}\right) = A \quad \Rightarrow \quad 4 - 0 = A \Rightarrow A = 4$$

Now $f(x) = \left[x^2\right]\sin(\pi x)$ is discontinuous when $x^2$ is integer but x is not integer

At $\quad x = \sqrt{A+5} = \sqrt{9} = 3 \Rightarrow$ continuous

$x = \sqrt{A} = \sqrt{4} = 2 \Rightarrow$ continuous

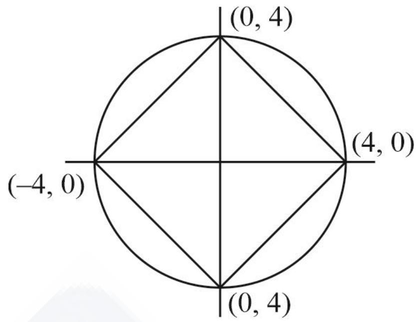
$x = \sqrt{A+21} = \sqrt{25} = 5 \Rightarrow$ continuous

$x = \sqrt{A+1} = \sqrt{5} \Rightarrow$ discontinuous

**16.(1)** Let $z = x + iy,\ x,\ y \in R$

$|x| + |y| = 4$

$|z| = \sqrt{(x)^2 + (y)^2} = \sqrt{|x|^2 + |y|^2} = \sqrt{(|x| + |y|)^2 - 2|x||y|}$




Now by A.M, G.M inequality

$$\frac{|x| + |y|}{2} \geq \sqrt{|x|\,|y|}$$

$\Rightarrow \quad 2 \geq \sqrt{|x|\,|y|} \Rightarrow |x||y| \leq 4$

$\therefore \quad 0 \leq |x|\,|y| \leq 4$

$\Rightarrow \quad -8 \leq -2|x|\,|y| \leq 0 \Rightarrow -8 + 16 \leq (|x| + |y|)^2 - 2|x||y| \leq 16 \Rightarrow 2\sqrt{2} \leq |z| \leq 4$

$\therefore \quad |z|$ can't be equal to $\sqrt{7}$

**17.(Bonus)**

$$\frac{\frac{(10)!}{2!3!5!} \times (4)! + \frac{10!}{2!3!5!} \times 4! + \frac{10! \times 4!}{2!2!2!3!3!2!}}{4^{10}}$$

$$= \frac{17 \times 945}{2^{15}}$$

None option is correct.

**18.(2)** $2\alpha = \frac{2b}{a} \Rightarrow \alpha = \frac{b}{a},\ \alpha^2 = \frac{5}{a}$

$\Rightarrow \quad \frac{b^2}{a^2} = \frac{5}{a}$

$\Rightarrow \quad b^2 = 5a \qquad \ldots\text{(i)}$

$\alpha + \beta = 2b \qquad \ldots\text{(ii)}$

$\alpha\beta = -10 \qquad \ldots\text{(iii)}$

$\alpha = \frac{b}{a}$ is a root of $x^2 - 2bx - 10 = 0$

$\Rightarrow \quad b^2 - 2ab^2 - 10a^2 = 0$

$5a - 10a^2 - 10a^2 = 0$

$\Rightarrow \quad a = \frac{1}{4},\ b^2 = \frac{5}{4}$

$\therefore \quad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2(\alpha\beta) = (2b)^2 + 20 = 4b^2 + 20 = 25$

**19.(1)** $2b = \frac{4}{\sqrt{3}} \Rightarrow b = \frac{2}{\sqrt{3}}$

We have, $y = mx \pm \sqrt{a^2m^2 + b^2}$

Is equation of tangent of slope m comparing with $y = -\frac{x}{6} + \frac{4}{3}$

$m = -\frac{1}{6},\ a^2m^2 + b^2 = \frac{16}{9}$

$\Rightarrow \quad \frac{a^2}{36} + \frac{4}{3} = \frac{16}{9} \Rightarrow a^2 = 16 \Rightarrow e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{4}{3 \times 16}} = \sqrt{\frac{11}{12}}$

**20.(3)**

| p | q | $\sim q$ | $p \wedge \sim q$ | $p \to (p \wedge \sim q)$ |
|---|---|---|---|---|
| T | T | F | F | F |
| T | F | T | T | T |
| F | T | F | F | T |
| F | F | T | F | T |

**SECTION – 2**

**21.(3)** $P$ is on line 1 ; $A(-1, 3, -1)$

$P$ is on line 2; $B(-3, -2, 1)$

$\overrightarrow{AB} = \begin{vmatrix} \hat{i} & j & k \\ -2 & -5 & 2 \\ 2 & 4 & 3 \end{vmatrix}$; $DR'$ of line 1 < 2, 4, 3 >

$\vec{n} = +23\hat{i} - 10j - 2k$

$\therefore$ eqn of plane : $23(x+1) - 10(y-3) - 2(z+1) = 0$

$23x - 10y - 2z + 51 = 0$

Other plane is $23x - 10y - 2z + 48 = 0$

$\therefore$ disc between them = $\dfrac{3}{\sqrt{633}}$ $\quad \therefore \quad K = 3$

**22.(30)** $\vec{b}.\vec{c} = 10$

$\Rightarrow |\vec{b}|\,|\vec{c}| \cos\frac{\pi}{3} = 10 \quad \Rightarrow 5.|\vec{c}|\frac{1}{2} = 10 \Rightarrow |\vec{c}| = 4$

Also, $\vec{a}.(\vec{b} \times \vec{c}) = 0$

$|\vec{a} \times (\vec{b} \times \vec{c})| = (\vec{a})|\vec{b} \times \vec{c}| \sin\frac{\pi}{2} = \sqrt{3}|\vec{b}|.\,4\sin\frac{\pi}{3} \times 1 = 30$

**23.(36)** The circles are $x^2 + (y-4)^2 = k$ & $(x-3)^2 + y^2 = 1$

Towards each other if $C_1C_2 = |r_1 \pm r_2|$

Where $C_1(0, 4), C_2(3, 0)$

$|C_1C_2| = \sqrt{k} + 1$ or $|\sqrt{k} - 1| \Rightarrow \quad 5 = \sqrt{k} + 1$ or $(\sqrt{k} - 1) = 5 \quad \Rightarrow \quad k = 16$ or $k = 36$

$\therefore$ Maximum value of $k$ = 36

**24.(51)** $\sum_{r=0}^{25} (4r+1)\,{}^{25}C_r = 4\sum_{r=0}^{25} r\,{}^{25}C_r + \sum_{r=0}^{25} {}^{25}C_r$

$= 4\sum_{r=0}^{25} 25\,{}^{24}C_{r-1} + 2^{25} = 100\; 2^{24} + 2^{25} = 2^{25}(50+1) = 51.2^{25} \Rightarrow k = 51$

**25.(14)** First common term = 23

Common difference = $7 \times 4 = 28$

Last term $\le 407$

$\Rightarrow \quad 23 + (n-1) \times 28 \le 407 \quad \Rightarrow \quad 28(n-1) \le 384$

$\Rightarrow \quad n \le 13.71 + 1 \quad \Rightarrow \quad n \le 14.71 \quad \Rightarrow n = 14$