Solutions to JEE Mains/Test Series - 9/IITJEE - 2013

[CHEMISTRY]

1.(C) By conservation of moles

$$\frac{PV}{RT} = \frac{P_1V_1}{RT} + \frac{P_2V_2}{RT}$$

$P \times 4 = 2 \times 1 + 3 \times 2$

$P = 2$ atm.

2.(C) Statement-II violates the inert pair effect.
Hence it is false.

3.(A) $KMnO_4$ will react with $FeSO_4$ only.

$1 \times \text{moles} = 5 \times 2 \times \frac{100}{1000}$

moles = 1

$$\chi_{FeSO_4} = 1/3$$

4.(D)

5.(D) Only Cl_2 is oxidsing among the given options

$\therefore \quad K_2MnO_4 \xrightarrow{Cl_2} KMnO_4$

6.(D) $\Delta S = nC_v \ln \frac{T_2}{T_1} + nR \ln \frac{V_2}{V_1}$

$= nC_v \ln 2 + nR \ln \frac{1}{2}$

$= (C_v - R) \ln 2$

7.(D) P-anthranilic acid has less acidic nature due to the +M effect of $-NH_2$ group.

8.(B)

$$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$$

$t = 0 \quad a \quad 3a$

$t = t_{eq} \quad \frac{a}{2} \quad \frac{3a}{2} \quad a$

$P_{NH_3} = \frac{a}{3a} P = \frac{P}{3}$.

9.(C) $4BF_3 + 3H_2O \longrightarrow H_3BO_3 + 3HBF_4$

10.(D) $[H^+] = \sqrt{Ka_1C_1 + Ka_2C_2}$ (Learn as a result)

$= \sqrt{1.8 \times 10^{-5} \times 0.01 + 6.3 \times 10^{-5} \times 0.01}$

$= \sqrt{8.1 \times 10^{-7}} = 9 \times 10^{-4}$.

11.(B) $Cr_2O_7^{2-}$ on reduction converts to Cr^{+3} which is green in colour.

12.(C) $\sqrt{n(n+2)} = 1.73 = \sqrt{3}$

$n\,(n+2) = 3$

n = no. of unpaired e^{-}s = 1

$v = 4s^2\ 3d^3$.

Hence Vanadium must exist in the form V^{4+} compound is VCl_4

13.(C) $CH_2 = CH - CH_2 - Cl \xrightarrow{AgNO_3} AgCl\downarrow + CH_2 = CH - \overset{\oplus}{C}H_2$ Allylic carbocation.

14.(A) $MgSO_4 + NH_4OH + Na_2HPO_4 \longrightarrow Mg(NH_4)PO_4 + Na_2SO_4 + H_2O$.

15.(C) $CH_3 - CH_2 - OH \xrightarrow{P + I_2} A \xrightarrow{Mg/Ether} B \xrightarrow{HCHO} C \xrightarrow{H_2O} D$

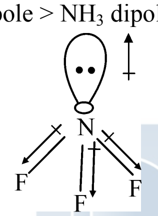
(A) is $CH_2 - CH_2 - I$ **(B)** is $CH_3 - CH_2 - Mg - I$

(C) is $CH_3 - CH_2 - CH_2 - O^{\ominus}\overset{\oplus}{M}gI$ **(D)** is $CH_3 - CH_2 - CH_2 - OH$

16.(B)

17.(B) $CH_3 - \underset{\|}{\overset{O}{C}} - CH_2 - \overset{O}{\underset{\|}{C}} - CH_2 - CH_3$ Most acidic hydrogen (Active methylene group)

18.(D) NF_3 dipole > NH_3 dipole.



19.(D) R.D.S. of cannizaro involves transfer of H^{-} ion to the carbonyl group.

20.(A) $R - \overset{O}{\overset{\|}{C}} - Z + Nu^{-} \longrightarrow R - \overset{O}{\overset{\|}{C}} - Nu + Z^{-}$

Leaving group capacity of Cl^{-} is maximum among the given options hence & fastest reaction when Z = Cl.

21.(C) Both (1) & (2) contain chiral carbon will undergo SN_2 reaction with alc. KCN, hence inversion.

22.(B) $\pi = iCRT$

$$0.75 = 2.47 \times \frac{\text{moles}}{2.5} \times 0.0821 \times 300$$

$$\text{Moles} = \frac{1}{30} = 0.03$$

23.(A) $E = \dfrac{CZ^2}{n^2}$

$$\frac{\varepsilon_H}{\varepsilon_{Be^{3+}}} = \frac{Z_H^2}{n_H^2}\,\frac{n_{Be^{3+}}^2}{Z_{Be^{3+}}^2} = \frac{1}{1^2} \times \frac{2^2}{4^2} = \frac{1}{4}$$

24.(D) Graphite: free e^{-} are spread out between the structure & thus graphite is conducting.

25.(D) V_1 = volume for complete neutralization of Na_2CO_3.

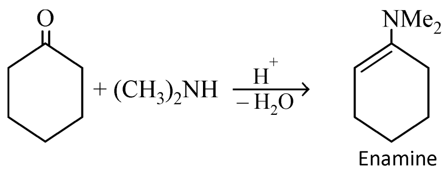
V_2 = Volume for complete neutralization of $NaHCO_3$.

$\dfrac{V_1}{2} = x$

$V_1 + V_2 = x + y$

Hence $V_2 = y - x$

26.(D) $Cl_{(g)} + e^- \longrightarrow Cl^-_{(g)}$ is exothermic, rest all endothermic.

27.(D) 

28.(C) Metal oxides an basic (or more ionic character $\approx$ more basic).

$\therefore$ TiO > VO > CrO > FeO

29.(C) $\frac{dx}{dt} = k[A]^2$

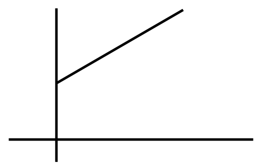
$\log\left(\frac{dx}{dt}\right) = \log k + 2\log[A]$

$y = 2x + \log k$

On comparing with y = mx + c.

Where m ≡ slope

c ≡ constant



30.(A) $4r = a\sqrt{2}$

$r = \frac{\sqrt{2}}{4}a$

Solutions to JEE Mains/Test Series - 9/IITJEE - 2013

[PHYSICS]

31.(C) A crystal structure is composed of a unit cell, a set of atoms arranged in a particular way; which is periodically repeated in three dimensions on a lattice. The spacing between unit cell in various directions is called its lattice parameters or constants. Increasing these lattice constant will increase or widen the band-gap (E_g), which means more energy would be required by electrons to reach the conduction band from the valence band. Automatically E_c and E_v decreases.

32.(D) $K = K_1 + K_2 + \frac{K_3K_4}{K_3 + K_4} = 50 + 30 + \frac{60 \times 30}{90} = 100\ N/m$

$T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{0.01}{100}} = \frac{\pi}{50}\ Hz.$

33.(B) $\frac{v_{t1}}{v_{t2}} = \frac{d_1 - \rho}{d_2 - \rho} \Rightarrow \frac{0.18}{v_{t2}} = \frac{20 - 2}{10 - 2} \Rightarrow v_{t2} = 0.08\ m/s$

34.(A) $\bar{C} \,||\, \bar{E} \times \bar{B}$ & $CB = E$ in magnitude.

As $\bar{C} = 3 \times 10^8\ \hat{i}$ and $\bar{E} = 720\ \hat{j} \Rightarrow \bar{B} = 2.4 \times 10^{-6}\ \hat{k}$

35.(C) $10{,}000 = 9500\left\{\frac{300 - 0}{300 - v}\right\} \Rightarrow 300 - v = 285 \Rightarrow v = 15\ m/s$

36.(B) Only electric force acts on the particle.

37.(C)

38.(A) Lowest frequency = *HCF* of 420 *Hz* & 315 *Hz* = 105 *Hz*.

39.(D) $\phi = BAN\ \sin \omega t$

$$\varepsilon = -\frac{d\phi}{dt} = -BAN\,\omega \cos \omega t \quad \Rightarrow \quad \varepsilon_m = BAN\omega$$

40.(B) $\varepsilon = -\frac{d\phi}{dt} = -\left(20t - 50\right) = -10v \quad at\ t = 3s$

41.(B) D_1 is not conducting but D_2 is conducting

$$\Rightarrow \quad i = \frac{12}{4+2} = 2A$$

42.(B) Energy of proton $= 8 \times 7.06 - 7 \times 5.6$

$= 17.28\ MeV$

43.(A) K.E. is maximum when P.E. is minimum $\Rightarrow \frac{dv}{dx} = 0 \Rightarrow x^3 - x = 0 \Rightarrow x = 0\ or\ x = \pm 1$

$\frac{d^2v}{dx^2} = 3x^2 - 1 < 0\ \ for\ x = 0 \quad \Rightarrow \quad x = 0$ is a point of maximum for V

and $\frac{d^2v}{dx^2} = 3x^2 - 1 > 0\ \ for\ x = \pm 1 \quad \Rightarrow \quad x = \pm 1$ are points of minima for V

$\Rightarrow$ minimum potential energy $= \frac{1}{4} - \frac{1}{2} = \frac{-1}{4}$J at $x = \pm 1$

$\Rightarrow$ maximum $KE = 2 - \left(\frac{-1}{4}\right) = \frac{9}{4} \quad \Rightarrow \quad \frac{9}{4} = \frac{1}{2} m v_{max}^2\ \&\ m = 1kg \Rightarrow \quad v_{max} = \frac{3}{\sqrt{2}}\ m/s$

44.(C) Let Q be total charge on the two spheres & let q_1 & q_2 be charges on A & B respectively in equilibrium. ($r_A = 1mm$, $r_B = 2mm$)

$\Rightarrow \quad q_1 + q_2 = Q$ and $\frac{q_1}{4\pi \in_0 r_A} = \frac{q_2}{4\pi \in_0 r_B} \Rightarrow q_1 = \frac{Q}{3}$ and $q_2 = \frac{2Q}{3}$.

$$E_1 / E_2 = q_1 / r_A^2 \ :\ q_2 / r_B^2 = 1/3 : \frac{2/3}{(2)^2} = 2 : 1$$

45.(A) $\mu_R < \mu_B \quad \Rightarrow D_1 < D_2 \qquad \because \left[\mu = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin A/2}\right]$

$\Rightarrow$ Ans. (A) or for small angled prism $D = A\left(\mu - 1\right)$

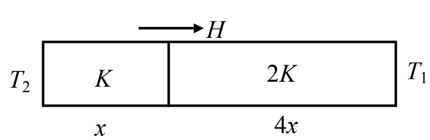
46.(D)

47.(C)

48.(A) $\frac{v^2}{R} \propto \frac{1}{R^n} \quad \Rightarrow \quad v \propto \frac{1}{R^{(n-1)/2}}$

$$T = \frac{2\pi R}{V} \propto R^{1 + \frac{n-1}{2}} \quad \Rightarrow \quad T \propto R^{\frac{n+1}{2}}$$

49.(D)



$\Rightarrow \quad H = \frac{T_2 - T_1}{\frac{x}{KA} + \frac{4x}{2KA}} = \frac{KA\left(T_2 - T_1\right)}{3x}$

$\Rightarrow \quad f = 1/3 \qquad \Rightarrow$ Ans. (D)

50.(B) $|\Delta P| = E/C - (-E/C) = 2E/C \qquad \Rightarrow$ Ans. (B)

51.(A) $u = v$ $\Rightarrow$ the object is placed at the centre of the equivalent mirror.

$$\frac{1}{F_m} = -2\left\{\left(\frac{1.5}{1} - 1\right)\left\{\frac{1}{\infty} - \frac{1}{-30}\right\}\right\} + \frac{1}{-30/2} \quad \Rightarrow \quad F_m = -10\,cm\,.$$

$R = |2F_m| = 20\ cm$

52.(C) $i = \dfrac{6}{\left[(2\|6) + 1.5\right]\|3} = 4A$

53.(A) $\dfrac{X}{Y} = \dfrac{20}{80}$; $\dfrac{4X}{Y} = \dfrac{x}{100 - x}$ $\Rightarrow$ $x = 50\ cm$

54.(B) Let ℓ : length of the wire ; R : radius in first case; r : radius in second case.

$\Rightarrow$ $2\pi R = \ell$ & $n(2\pi r) = \ell$

$$B = \frac{\mu_0 i}{2R} = \frac{\mu_0 \pi i}{\ell}$$

$$B' = \frac{n\,\mu_0 i}{2r} = \frac{n^2 \mu_0 \pi i}{\ell} = n^2 B$$

55.(B) $T = 2\pi\sqrt{\dfrac{I}{MB}}$ where I: moment of Inertia $= \dfrac{m\ell^2}{12}$

M : pole strength $\times$ ℓ

$$I' = \frac{m(\ell/s)^2}{12} = I/9\ ;\ M' = (\text{pole strength remain same}) \times 3 \times \ell/3M$$

$\Rightarrow$ $T' = 2\pi\sqrt{\dfrac{I/9}{M.B}} = \dfrac{T}{3} = 2/3\ sec$

56.(D) As $P.d$ across L & C are out of phase.

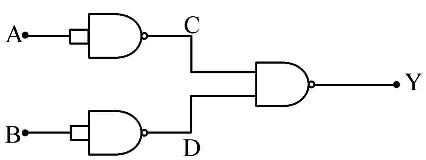
57.(B) induced current $= \dfrac{(\text{change influx}) \times \text{no of turns}}{\text{total resistance} \times \text{time}} = \left\{\dfrac{(W_2 - W_1)}{(R + 4R)} \times n\right\} \div t = -\dfrac{n(W_2 - W_1)}{5RT}$

58.(C) $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{L'(2C)}}$ $\Rightarrow$ $L' = L/2$

59.(B) $\varepsilon = \dfrac{1}{2}\Delta\omega\ell^2 = \dfrac{1}{2} \times \left(0.2 \times 10^{-4}\right) \times 5 \times (1)^2 = 5 \times 10^{-5} V$

$= 50\ \mu v$

60.(A)



A	C
0	1
1	0

B	D
0	1
1	0

A	B	C	D	Y
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

$\Rightarrow$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

$\Rightarrow$ truth table of OR gate.

Solutions to JEE Mains/Test Series - 9/IITJEE - 2013

[MATHEMATICS]

61.(D) For the graph to touch x-axis, the polynomial $y = x^2 - 2px + p + 1$ would have to be a perfect square. Therefore, $p^2 = p + 1$, or $p^2 - p - 1 = 0$, $p = \left(1 \pm \sqrt{5}\right)/2$.

62.(C) $Z_1 = \left(8\sin\theta + 7\cos\theta\right) + i\left(\sin\theta + 4\cos\theta\right)$

$Z_2 = \left(\sin\theta + 4\cos\theta\right) + i\left(8\sin\theta + 7\cos\theta\right)$

Hence, $\left.\begin{matrix} Z_1 = x + iy \\ Z_2 = y + ix \end{matrix}\right]$ where $x = \left(8\sin\theta + 7\cos\theta\right)$ and $y = \left(\sin\theta + 4\cos\theta\right)$

$Z_1 . Z_2 = \left(xy - xy\right) + i\left(x^2 + y^2\right)$

$= a + ib \Rightarrow a = 0\ ;\ b = x^2 + y^2$

Now, $x^2 + y^2 = \left(8\sin\theta + 7\cos\theta\right)^2 + \left(\sin\theta + 4\cos\theta\right)^2$

$= 65\sin^2\theta + 65\cos^2\theta + 120\sin\theta\cos\theta$

$= 65 + 60\sin 2\theta$

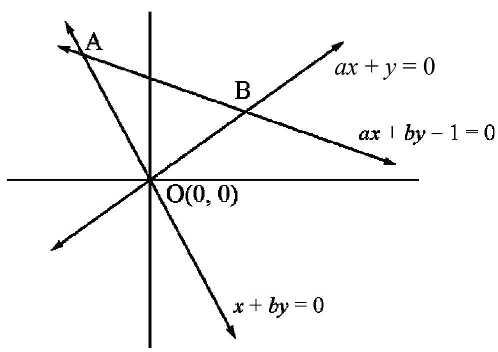
Hence, $\left(a + b\right)_{max} = 125$

63.(C) $|2\left(\bar{a} \times \bar{b}\right)| + 3\left(\bar{a} . \bar{b}\right)$

$= 2|\bar{a}||\bar{b}|\sin\theta + 3|\bar{a}||\bar{b}|\cos\theta$, where θ is angle between $\bar{a}$ and $\bar{b}$

$= 12\sin\theta + 18\cos\theta \le \sqrt{12^2 + 18^2} = 6\sqrt{13}$

64.(B) Line $ax + y = 0$ and $x + by = 0$ intersect at O(0, 0).



If AB subtends right angle at O(0, 0), then $ax + y = 0$ and $x + by = 0$ are perpendicular.

$\Rightarrow \quad \left(-a\right)\left(-\dfrac{1}{b}\right) = -1 \quad \Rightarrow \quad a + b = 0$

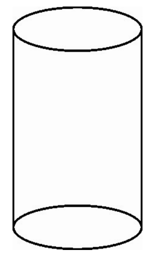
65.(D) The first two columns of first determinant are same as first two rows of second. Hence, transpose the second. Add the two determinants and use $C_1 \to C_1 + C_3$. It gives D = 0.

66.(D) $\left(\dfrac{dr}{dt}\right) = c$ and $h = ar + b$; also $\left(\dfrac{dh}{dt}\right) = 3\left(\dfrac{dr}{dt}\right)$ (given)

$\therefore \quad a\frac{dr}{dt} = 3\frac{dr}{dt} \Rightarrow a = 3$

Hence, $h = 3r + b$

When $r = 1$; $h = 6 \Rightarrow 6 = 3 + b \Rightarrow b = 3$



$\therefore \quad h = 3(r+1)$

$$V = \pi r^2 h = 3\pi r^2 (r+1) = 3\pi (r^3 + r^2)$$

$$\frac{dV}{dt} = 3\pi (3r^2 + 2r)\frac{dr}{dt}$$

where $r = 6$; $\left(\frac{dV}{dt}\right) = 1\ cc/sec$

Therefore, $1 = 3\pi(108+12)\left(\frac{dr}{dt}\right) \Rightarrow 360\pi\left(\frac{dr}{dt}\right) = 1$

Again when $r = 36, \left(\frac{dV}{dt}\right) = n$

$$n = 3\pi\left(3(36)^2 + 2.36\right)\left(\frac{dr}{dt}\right)$$

$$n = 3\pi \,.\, 36(110)\,.\,(1/360\pi)$$

$$n = 33$$

67.(A) $L = \lim_{x\to 0}\left(\frac{\sin 3x}{x^3} + \frac{a}{x^2} + b\right)$

$$= \lim_{x\to 0}\frac{\sin 3x + ax + bx^3}{x^3}$$

$$= \lim_{x\to 0}\frac{3\frac{\sin 3x}{3x} + a + bx^2}{x^2}$$

For existence of limit $3 + a = 0$

$\Rightarrow \quad a = -3$

$\therefore \quad L = \lim_{x\to 0}\frac{\sin 3x - 3x + bx^3}{x^3} = 27\lim_{t\to 0}\frac{\sin t - t}{t^3} + b = 0\ (3x = t)$

$= -\frac{27}{6} + b = 0$ (Apply LH rule to get $\lim_{t\to 0}\frac{\sin t - t}{t^3} = \frac{-1}{6}$ $\Rightarrow$ $b = \frac{9}{2}$

68.(C) Given integral

$$= \int_0^1 \frac{dx}{(x+\cos\alpha)^2 + (1-\cos^2\alpha)}$$

$$= \int_0^1 \frac{dx}{(x+\cos\alpha)^2 + \sin^2\alpha}$$

$$= \frac{1}{\sin\alpha}\left|\tan^{-1}\frac{x+\cos\alpha}{\sin\alpha}\right|_0^1$$

$$= \frac{1}{\sin\alpha}\left[\tan^{-1}\frac{1+\cos\alpha}{\sin\alpha} - \tan^{-1}\frac{\cos\alpha}{\sin\alpha}\right]$$

$$= \frac{1}{\sin\alpha}\left[\tan^{-1}\cot\frac{\alpha}{2} - \tan^{-1}(\cot\alpha)\right]$$

$$= \frac{1}{\sin\alpha}\left[\tan^{-1}\tan\left(\frac{\pi}{2}-\frac{\alpha}{2}\right) - \tan^{-1}\tan\left(\frac{\pi}{2}-\alpha\right)\right]$$

$$= \frac{1}{\sin\alpha}\left[\left(\frac{\pi}{2}-\frac{\alpha}{2}\right) - \left(\frac{\pi}{2}-\alpha\right)\right] = \frac{\alpha}{2\sin\alpha}$$

69.(B) $\sqrt{\log_2 x - 1} - \frac{3}{2}\log_2 x + 2 > 0 \, (x > 0)$

$\Rightarrow \quad \sqrt{\log_2 x - 1} - \frac{3}{2}(\log_2 x - 1) + \frac{1}{2} > 0$

Let $\quad \sqrt{\log_2 x - 1} = t \geq 0$ **. . . .(i)**

$\Rightarrow \quad \log_2 x \geq 1 \;\Rightarrow\; x \geq 2$

$\therefore \quad t - \frac{3}{2}t^2 + \frac{1}{2} > 0$

$\Rightarrow \quad 2t - 3t^2 + 1 > 0 \;\Rightarrow\; 3t^2 - 2t - 1 < 0$

$\Rightarrow \quad -\frac{1}{3} < t < 1$ **. . . .(ii)**

From (i) and (ii), $0 \leq t < 1$

$0 \leq \sqrt{\log_2 x - 1} < 1$

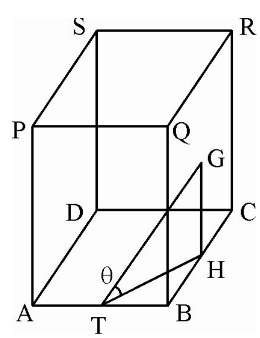
$0 \leq \log_2 x - 1 < 1$

$1 \leq \log_2 x < 2$

$2 \leq x < 4$

70.(A) Let H be the midpoint of BC since $\angle TBH = 90°, TH^2 = BT^2 + BH^2$

$= 5^2 + 5^2 = 50$



Also $\quad \angle THG = 90°, TG^2 = TH^2 + GH^2 = 50 + 25$

$= 75$

Let θ be the required angle of elevation of G at T.

Then $\quad \sin\theta = \frac{GH}{TG} = \frac{5}{5\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \theta = \sin^{-1}\frac{1}{\sqrt{3}}$

71.(D) $f(x)\begin{cases} \frac{\tan[x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases} = \begin{cases} \frac{\tan[x]}{[x]}, & x \in (-\infty, 0) \cup [1, \infty) \\ 0, & x \in [0, 1) \end{cases}$

L.H.L $= \lim\limits_{x \to 0-} f(x) = \lim\limits_{x \to 0} \frac{\tan[x]}{[x]} = \lim\limits_{h \to 0} \frac{\tan[-h]}{[-h]} = \frac{\tan(-1)}{(-1)} = \tan 1$

R.H.L. $= \lim\limits_{x \to 0+} f(x) = \lim\limits_{x \to 0} 0 = 0$

Since L.H.L. $\neq$ R.H.L. Therefore $\lim\limits_{x \to 0} f(x)$ does not exist.

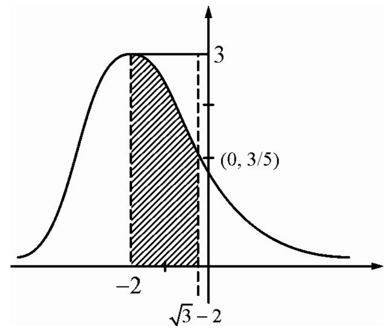
72.(B) $\left({}^{10}C_0\right)+\left({}^{10}C_0+{}^{10}C_1\right)+\left({}^{10}C_0+{}^{10}C_1+{}^{10}C_2\right)+\ldots\left({}^{10}C_0+{}^{10}C_1+{}^{10}C_2+{}^{10}C_9\right)$

$$=10\,{}^{10}C_0+9\,{}^{10}C_1+8\,{}^{10}C_2+\ldots+{}^{10}C_9$$

$$={}^{10}C_1+2\,{}^{10}C_2+3\,{}^{10}C_3+\ldots+10\,{}^{10}C_{10}$$

$$=\sum_{r=1}^{10} r\,{}^{10}C_r=10\sum_{r=1}^{10}{}^{9}C_{r-1}=10\cdot 2^9$$

73.(D) $y'=\dfrac{-2(x+2)}{\left(x^2+4x+5\right)^2}=0 \Rightarrow x=-2$;



$$\int\frac{3}{x^2+4x+5}\,dx=\int\frac{3}{(x+2)^2+1}\,dx=3\tan^{-1}(x+2)+C$$

Hence A = 3 and B = 2

Hence $A \neq B$

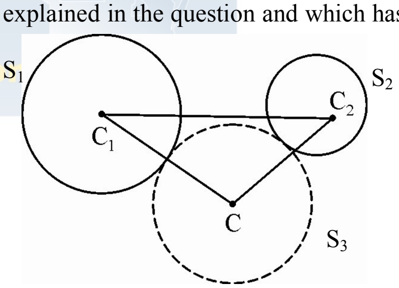
Area $=\displaystyle\int_{-2}^{\sqrt{3}-2} 3f(x)\,dx=3\tan^{-1}(x+2)\Big]_{-2}^{\sqrt{3}-2}=3\tan^{-1}\left(\sqrt{3}\right)=\pi$

Range is (0, 1]

Graph of $y=3f(x)$ is as shown. Turning point is $x=-2$

74.(B) Circles S_1,and S_2 with centres C_1 and C_2 have radius r_1 and r_2, respectively

Let S_3 be the variable circle which touches the given two circles as explained in the question and which has centre C and radius *r*.



Now $CC_2=r+r_2$ and $CC_1=r_1+r$

Hence $CC_1-CC_2=r_1-r_2$ (= constant)

The locus of C is hyperbola whose focii are C_1 and C_2.

75.(B) Let the tangents interest at P(h, k) then

According to the question

$h\cos\theta+k\sin\theta=a$

$h\cos\left(\theta+\dfrac{\pi}{3}\right)+k\sin\left(\theta+\dfrac{\pi}{3}\right)=a$

Eliminating θ from the above equations we get

$h^2+k^2=\dfrac{4a^2}{3}$

76.(B) $\tan^{-1}1=\dfrac{\pi}{4}<1$

1 Radian $=57°17'44.8''$ for which $\sin 1>\cos 1$ but both values are less than 1.

Obviously, *tan* 1 > 1

Hence greatest value is *tan* 1

77.(D) We know that $P\left(\dfrac{A}{B}\right)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{P(A)+P(B)-P(A\cup B)}{P(B)}$

Since $P(A\cup B)<1$, therefore $-P(A\cup B)>-1$

$\Rightarrow \quad P(A)+P(B)-P(A\cup B)>P(A)+P(B)-1$

$\Rightarrow \quad \dfrac{P(A)+P(B)-P(A\cup B)}{P(B)} > \dfrac{P(A)+P(B)-1}{P(B)} \quad \Rightarrow \quad P\left(\dfrac{A}{B}\right) > \dfrac{P(A)+P(B)-1}{P(B)}$

Thus, choice (A) is correct.
Choice (B) holds good.
Choice (C) is also correct since

$$P(A\cup B) = P(A)+P(B)-P(A\cap B)$$
$$= P(A)+P(B)-P(A)P(B) \text{ If A, B are independent}$$
$$= 1-(1-P(A))(1-P(B)) = 1-P(\overline{A})P(\overline{B})$$

Choice (D) is obviously not true because $P(A\cup B) = P(A)+P(B)$

78.(B) $f(x) = \int \dfrac{x^2 dx}{(1+x^2)\left(1+\sqrt{1+x^2}\right)}$

Let $\quad x = tan\,\theta \;\Rightarrow\; dx = sec^2\theta\, d\theta$

$\Rightarrow \quad f(x) = \int \dfrac{x^2 dx}{(1+x^2)\left(1+\sqrt{1+x^2}\right)}$

$$= \int \frac{tan^2\theta\, sec^2\theta\, d\theta}{sec^2\theta(1+sec\,\theta)} = \int \frac{tan^2\theta\, d\theta}{1+sec\,\theta}$$

$$= \int \frac{sin^2\theta\, d\theta}{cos\,\theta(1+cos\,\theta)}$$

$$= \int \frac{1-cos^2\theta\, d\theta}{cos\,\theta(1+cos\,\theta)}$$

$$= \int \frac{(1-cos\,\theta)\,d\theta}{cos\,\theta}$$

$$= \int sec\,\theta\, d\theta - \int d\theta$$

$$= log\left(x+\sqrt{1+x^2}\right) - tan^{-1}x + c$$

Given $\quad f(0) = 0$

$\Rightarrow \quad 0 = log\,1 - 0 + c \quad \Rightarrow \quad c = 0$

$\Rightarrow \quad f(1) = log\left(1+\sqrt{1+1^2}\right) - tan^{-1}(1)$

$$= log\left(1+\sqrt{2}\right) - \frac{\pi}{4}$$

79.(D) Let $I = \displaystyle\int_1^e \frac{1+x^2 lnx}{x+x^2\, ln\,x}\, dx$

$$\frac{1+x^2\, ln\,x}{x+x^2\, ln\,x} = \frac{(1+x\,ln\,x)-\left(x\,ln\,x - x^2\, ln\,x\right)}{x(1+x\,ln\,x)}$$

$$= \frac{1}{x} - \frac{(ln\,x - x\,ln\,x)}{1+x\,ln\,x} = \frac{1}{x} + \frac{(1+x\,ln\,x - ln\,x - 1)}{1+x\,ln\,x}$$

$$= \left(\frac{1}{x}+1\right) - \left(\frac{1+ln\,x}{1+x\,ln\,x}\right)$$

$$I=\int_1^e\left(\frac{1}{x}+1\right)dx-\int_1^e\frac{1+ln\,x}{1+x\,ln\,x}dx$$

$$=\left[ln\,x+x\right]_1^e-\left[ln\left[1+x\,ln\,x\right]\right]_1^e$$

$$=e-ln(1+e)$$

80.(C) Since $\lim_{x\to1^-}g(x)=\lim_{x\to1^+}g(x)=1$ and $g(1)=0$

So, $g(x)$ is not continuous at x = 1 but $\lim_{x\to1}g(x)$ exists.

We have, $\lim_{x\to1^-}f(x)=\lim_{h\to0}f(1-h)=\lim_{h\to0}[1-h]=0$

and $\lim_{x\to1^+}f(x)=\lim_{h\to0}f(1+h)=\lim_{h\to0}[1+h]=1$

So, $\lim_{x\to1}f(x)$ does not exist and so $f(x)$ is not continuous at $x=1$

Now $gof(x)=g(f(x))=g([x])=0 \quad \forall\ x,\ y\in R$

So gof is continuous for all values of 'x'

We have, $fog(x)=f(g(x))$

$$=\begin{cases}f(0), & x\in Z\\ f(x^2), & x\in R-Z\end{cases}$$

$$=\begin{cases}0, & x\in Z\\ [x^2], & x\in R-Z\end{cases}$$

Which is clearly not continuous.

81.(A) We have $f(x)=\begin{cases}ax^2+b, & x<-1\\ bx^2+ax+4, & x\ge-1\end{cases}$

$$f(x)=\begin{cases}2ax, & x<-1\\ 2bx+a, & x\ge-1\end{cases}$$

Since $f(x)$ is differentiable at $x=-1$, it is continuous at $x=-1$ and hence

$$\lim_{x\to-1-}f(x)=\lim_{x\to-1+}f(x)$$

$\Rightarrow \quad a+b=b-a+4 \Rightarrow a=2$ and also, $\lim_{x\to-1-}f'(x)=\lim_{x\to-1+}f'(x)$

$\Rightarrow \quad -2a=-2b+a \qquad \Rightarrow \quad 3a=2b\Rightarrow b=3\ (\because a=2)$

Hence $a=2,\ b=3$

82.(B) On differentiating the given equal w.r.t. x, we get

$$xy'+y\,.\,1-\frac{1}{y}y'=0 \quad\Rightarrow\quad xyy'-y'+y^2=0 \quad\Rightarrow\quad (xy-1)y'+y^2=0$$

On differentiating agains w.r.t. x, we get

$$(xy-1)y''+y'(xy'+y\,.\,1)+2yy'=0$$

$\Rightarrow \quad x\left(yy''+y'^2\right)-y''+3yy'=0$

$\therefore \quad k=3$

83.(A) Given $x-y+2z=5,\ 3x+y+z=6$

Let $z=\lambda$

Then $x-y=5-2\lambda$, and $3x+y=6-\lambda$

Solving these two equations, $4x = 11 - 3\lambda$ and $4y = 4x - 20 + 8\lambda = -9 + 5\lambda$

Then equation of the line is $\frac{4x-11}{-3} = \frac{4y+9}{5} = \frac{z-0}{1}$

84.(A) $A = \frac{sin^3 x}{1 + cos\, x} + \frac{cos^3 x}{1 - sin\, x}$

$$= \frac{\left(sin^3 x + cos^3 x\right) + \left(cos^4 x - sin^4 x\right)}{\left(1 + cos\, x\right)\left(1 - sin\, x\right)} = \frac{\left\{\left(sin\, x + cos\, x\right)\left(1 - sin\, x\, cos\, x\right) + \left(cos\, x + sin\, x\right)\left(cos\, x - sin\, x\right)\right\}}{\left(1 + cos\, x\right)\left(1 - sin\, x\right)}$$

$$= \frac{\left\{\left(sin\, x + cos\, x\right)\left(1 - sin\, x\, cos\, x + cos\, x - sin\, x\right)\right\}}{1 + cos\, x - sin\, x - sin\, x\, cos\, x} = sin\, x + cos\, x = \sqrt{2}\ cos\left(\pi/4 - x\right)$$

85.(A) As $A^2 = O,\ A^k = O\ \forall\ k \geq 2$

Thus, $\left(A + I\right)^{50} = I + 50A$

$\Rightarrow$ $\left(A + I\right)^{50} - 50A = I$ $\Rightarrow$ $a = 1, b = 0, c = 0, d = 1$

86.(D) 2 can be taken in 2 way (2^0 or 2^2)
3 can be taken in 3 ways (3^0 or 3^2 or 3^4)
Similarly, 5 can be taken in 4 ways (5^0 or 5^2 or 5^4 or 5^6)
and 7 can be taken in 5 ways (7^0 or 7^2 or 7^4 or 7^6 or 7^8)
Hence, the total divisors that are perfect squares = 2 . 3. 4. 5 = 120.

87.(C) The total number of ways in which 3 integers can be chosen from first 20 integers is ${}^{20}C_3$.
The product of three integers will be even if at least one of the integers is even. Therefore, the required probability = 1 – Probability that none of the three integers is even.

$$= 1 - \frac{{}^{10}C_3}{{}^{20}C_3} = 1 - \frac{2}{19} = \frac{17}{19}$$

88.(A) $\left({}^{400}C_4 + {}^{400}C_3\right) + {}^{401}C_3 + \ldots + {}^{500}C_3$

$$= \left({}^{401}C_4 + {}^{401}C_3\right) + {}^{402}C_3 + \ldots + {}^{500}C_3$$

$$= \left({}^{500}C_4 + {}^{500}C_3\right) = {}^{501}C_4$$

89.(Cancel) Statement 2 : $\vec{r} \times \left(i + 2\hat{j} - 3\hat{k}\right)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 2 & -3 \end{vmatrix}$$

$$= \hat{i}\left(-3y - 2z\right) - \hat{j}\left(-3x - z\right) + \hat{k}\left(2x - y\right)$$

$\therefore$ $-3y - 2z = 2,\ -3x - z = 0,\ 2x - y = -1$

$\Rightarrow$ $z = -3x$

$\Rightarrow$ $-3y - 2\left(-3x\right) = 2$ or $6x - 3y = 2$ or $2x - y = \frac{2}{3}$

$2x - y = -1$ and $2x - y = 2/3$ are parallel plane

So, $\bar{r} \times \left(\hat{i} + 2\hat{j} - 3\hat{k}\right) = 2\hat{i} - \hat{k}$ is not a straight line

Statement 1 : $\hat{r}\times\left(2\hat{i}-\hat{j}+3\hat{k}\right)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x & y & z\\ 2 & -1 & 3\end{vmatrix}$

$=\hat{i}\left(3y+z\right)-\hat{j}\left(3x-2z\right)+\hat{k}\left(-x-2y\right)$

$\therefore$ $3y+z=3,\ 3x-2z=0,\ -x-2y=1$

$3x-2\left(3-3y\right)=0$

$\Rightarrow$ $3x+6y=6 \Rightarrow x+2y=2$

Now, $x+2y=-1,\ x+2y=2$ are parallel planes

Therefore, $\vec{r}\times\left(2\hat{i}-\hat{j}+3\hat{k}\right)=3\hat{i}+\hat{k}$ is not a straight line.

Both the statements are false.

90.(D) Statement 1 is false.

$\because$ Sum of the length of any two sides of a triangle is greater than the length of the third side. Statement 2 is true.

$\because$ If $a^2+c^2-b^2<0$

then $\cos B<0 \Rightarrow B$ is obtuse

ANSWERS - JEE Mains/Test Series - 9/IITJEE-2013 (ACEG)

CHEMISTRY

1	2	3	4	5	6	7	8	9	10
C	C	A	D	D	D	D	B	C	D
11	12	13	14	15	16	17	18	19	20
B	C	C	A	C	B	B	D	D	A
21	22	23	24	25	26	27	28	29	30
C	B	A	D	D	D	D	C	C	A

PHYSICS

31	32	33	34	35	36	37	38	39	40
C	D	B	A	C	B	C	A	D	B
41	42	43	44	45	46	47	48	49	50
B	B	A	C	A	D	C	A	D	B
51	52	53	54	55	56	57	58	59	60
A	C	A	B	B	D	B	C	B	A

MATHEMATICS

61	62	63	64	65	66	67	68	69	70
D	C	C	B	D	D	A	C	B	A
71	72	73	74	75	76	77	78	79	80
D	B	D	B	B	B	D	B	D	C
81	82	83	84	85	86	87	88	89	90
A	B	A	A	A	D	C	A	Cancel	D